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# ABSTRACT

# ON THE

# MECHANISM

# OF THE MOTIONS OF

# FLOATING BODIES.

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THis little Treatiſe contains more knowledge in the art of ſhip-building then any book hitherto publiſhed that I have met with: the principles the author proceeds upon are juſt and true; I have verified them by a number of experiments, and they agree exactly with the calculations he gives: but what proved moſt ſatisfactory to me, was their anſwering perfectly well when put into practice, in ſeveral line of battle ſhips and frigates, that I built whilſt I was in Ruſſia.

CHARLES KNOWLES.

# THE

# MECHANISM

## Of the MOTIONS of

# FLOATING BODIES.

§. I. EVERY floating body, whether in a river, canal, lake, or ſea, is ſuſtained by the efforts of the preſſures of the water alone, againſt that part thereof which is immerged, whence:

1. Theſe efforts of the preſſures of the water, are its only ſupports.

2. Their whole united force is always equal to its gravity, when it is quite free from any external force: or to the whole gravity of its heavy parts, together with the forces applied thereto. When it floats its ſupports are neceſſarily equal to its tendency towards the centre of the earth.

3. Their common direction is vertical. They could not form its ſupports, but by acting in a direction oppoſite to its tendency towards the earth's centre.

4. Whilſt their whole force and their vertical direction are alſo invariably determined, they can no ways be altered, either by the figure of the part immerged, or by the homogeneity, or the heterogeneity of its internal parts.

5. Their centre of effort is the point of ſuſtentation of the floating body; it is the centre of its ſupports.

6. Its centre of gravity is always in the perpendicular of its point of ſuſtentation, when being free from all external force, it remains fixt in a certain poſition; and the common centre of gravity of its heavy parts, and of the forces applied to it, is likewiſe always in the perpendicular of its point of ſuſtentation, when receiving the impreſſion of thoſe forces, it remains fixed in a certain poſition. They are two equal forces which act oppoſitely, and reſt in equilibrio.

§. II. The point of its capacity, which is immediately and perpendicularly above its point of ſuſtentation, in this latter point meets with an unſurmountable obſtacle to its tendency towards the earth's centre: all the other parts being by their coheſions reſtrained to certain diſtances from the former point, are kept ſuſpended thereby. It is conſequently the point of ſuſpenſion of the floating body.

§. III. The centre of its ſuſtentation being the centre of effort of the preſſures of the water againſt its immerged part, and its centre of ſuſpenſion the point of its capacity which is immediately and perpendicularly above its point of ſuſtentation; it is an unavoidable conſequence that the mechaniſm of its motions is the ſame with that of the motions it would have, if at the time of the application of the ſame forces, it were ſuſpended in air by the ſame point wherewith it is ſuſpended in water.

§. IV. When it inclines.

1. Its line of ſuſtentation is the parallel to the horizon which paſſes through its actual point of ſuſtentation, and is at the ſame time parallel to the length of the ſide on which it inclines.

2. Its axis of inclination is the parallel to the horizon which paſſes through its point of actual ſuſpenſion, and is all along incumbent on its line of ſuſtentation.

3. All its parts deſcribe, during its inclination, arcs or portions of curves, parallel to one another, whoſe centre or focus is conſtantly in its axis of inclination, and whoſe ſurfaces are perpendicular to that axis.

§. V. As it is at liberty to incline on all its ſides, it may be ſaid to have as many poſſible lines of ſuſtentation and axes of inclination, as we can conceive diameters in two horizontal circles, which would be concentrick to its point of ſuſtentation and its point of ſuſpenſion.

§. VI. When it turns on itſelf in tacking, the axis of ſuch circular motion is the perpendicular of its point of ſuſtentation, and all its parts deſcribe then arcs concentrick to that perpendicular.

§. VII. Any force whoſe direction is perpendicular to the horizon,

1. Augments its gravity, when it is incumbent thereon, or diminiſhes it, when its action tends to elevate it.

2. In producing or preventing its inclination, it acts on a leaver equal to the diſtance between the line of its direction, and the vertical plane of the axis of the inclination which it produces or prevents.

§. VIII. Any force whoſe direction is horizontal, acts,

1. To move it horizontally.

2. It augments its gravity, when the angle of incidence of its line of direction on the ſurface of the body is acute, and below that line; and it diminiſhes it when that angle is acute, and above it: but when the ſaid angle of incidence is a right one, it neither augments nor diminiſhes it.

3. In producing or preventing its inclination, it acts on a leaver equal to the diſtance between its line of direction, and the horizontal plane of the axis of the inclination which it aſſiſts or obſtructs.

4. It tends to make it tack, or to obſtruct it in doing ſo on a leaver equal to the diſtance between its line of direction, and the perpendicular of the actual point of ſuſtentation of the floating body.

But it is neceſſary to obſerve, that we are not to admit any force of a leaver in the quantity wherewith its gravity and the velocity of its horizontal tranſlations are augmented or diminiſhed by the forces which are applied to it.

§. IX. Any force that acts on it, has,

1. Its abſolute power equal to the diſtance between the point to which it applies its centre of effort, and the axis of the motion which it produces or prevents.

2. Its relative power equal to the length of the leaver on which it acts, and which I have determined before in §. VII. and §. VIII.

3. Its abſolute moment equal to the product of itſelf, multipled by its abſolute power.

4. Its relative moment equal to the product of itſelf, multiplied by its relative power.

§. X. When ſeveral forces act upon it at the ſame time,

1. It does not remain fixed in one determinate inclination, except when the forces which produce its inclination, and thoſe which prevent it, are in equilibrio about the actual axis of its inclination.

2. It does not remain conſtantly turned towards the ſame quarter, except when the forces which act thereon, to turn it in one ſenſe, are in equilibrio with thoſe which tend to turn it in an oppoſite ſenſe, about the perpendicular of its actual point of ſuſtentation.

3. It does not acquire an uniform velocity in its horizontal tranſlations, but when thoſe different equilibriums are accompliſhed.

§. XI. When ſeveral forces which have different directions, do act upon it at the ſame time, we make uſe of the ordinary rules of the compoſition of motions, to determine the axis of the inclination which they produce, and the line of direction of its horizontal tranſlation.

We make uſe of the ſame rules, for reſolving a force whoſe direction is at the ſame time inclined to its length and to the horizon.

§. XII. To determine the perpendicular of its point of ſuſtentation.

1. I conſider that its immerged part, and the bulk of water which it diſplaces, are perfectly equal and ſimilar.

2. I ſuppoſe the immerged part to be compounded of homogene parts, as is the bulk of water which it diſplaces, in which caſe their centres of gravity are homologous.

3. I divide them both in my mind, into infinitely ſmall vertical filaments: and I obſerve,

That all the filaments, into which I have divided the diſplaced bulk of water, are in equilibrio about the perpendicular of its centre of gravity.

That the weight of each of them is perfectly equal to the force of the preſſure of the water, againſt the baſe of the vertical homologous filament of the immerged part.

That ſo, all the efforts of the preſſure of the water againſt the baſes of the verticle filaments, into which I have divided the immerged part of the floating body, are likewiſe in equilibrio about the perpendicular of its centre of gravity.

§. XIII. To determine the height of the ſame point of ſuſtentation: I ſuppoſe, firſt, that the body floats on ſtill water; that its parts above the ſurface of the water are annihilated; that its whole bulk conſiſts of parts homogene, inflexible, and light as air; and laſtly, that it is governed by an external force, equal to the weight of the whole floating body, and applied on the plane of its flotation, in ſuch ſort that this plane be on a level with the ſurface of the water.

I then in my imagination divide this immerged part into infinitely thin horizontal *laminæ* or ſlices. I ſeize the firſt or loweſt ſlice by its ſides; and diſengaging at once the immerged part from the external force, I obſerve.

1. That all the ſlices above it are, by the preſſure of the water, repelled together towards the ſurface of the water.

2. That to keep this firſt ſlice in its place, there is need only of a force equal to the weight of the bulk of water that it diſplaces.

Then the firſt ſlice being in its former place, I bring the immerged part under water; I ſeize again the ſides of the ſecond ſlice; I diſengage the immerged part from the external force, and I obſerve again.

1. That all the ſlices above it are, by the preſſures, repelled together towards the ſurface of the water.

2. That in caſe I bring the ſecond into an horizontal ſituation, and ſeize its ſides; the firſt is by the preſſures of the water repelled againſt its baſe; and that in order to retain them in their place, there is required a force equal to the weight of the bulk of water which theſe two ſlices diſplace. This makes me judge that all the efforts of the preſſures of the water againſt theſe two ſlices, are united in the ſecond.

I repeat this experiment, from ſlice to ſlice, quite up to that which forms the plane of flotation of the immerged part; and I obſerve, that in caſe I bring the ſlice which I ſeize, into an horizontal ſituation, all the ſlices under it are, by the preſſures of the water, repelled againſt it: and that to retain it in the water, there is requiſite a force equal to the bulk of the water diſplaced by this ſlice, and by thoſe under it; which makes me judge that all the efforts of the preſſures of the water againſt theſe ſeveral ſlices, are always united in that which is uppermoſt. In a word, that the infinitely thin one, which forms the plane of flotation, is the only one which requires a force equal to the weight of the whole bulk of water, diſplaced by the immerged part of the floating body, to keep it under water, or on a level with the ſurface of the water: whence I conclude, that all the efforts of the preſſures of the water againſt the immerged part, are united in the plane of flotation; and conſequently, that it is in this plane that their centre of effort reſides.

§. XIV. But the floating body has for its point of ſuſtentation, the centre of effort of the preſſures of the water, (§. I. preced.) and this centre of effort reſides in the perpendicular of the centre of gravity of its immerged part, (§. XII. preced.) and in the plane of its flotation (§. XIII. preced.) The floating body therefore, when it floats on ſtill water, has always for its point of ſupport, the point of ſection of the vertical of the centre of gravity of the part of it actually immerged, by the plane of its actual flotation.

It has likewiſe for the point of ſuſpenſion, the point of its capacity, which is immediately and perpendicularly above its point of ſuſtentation.

(§. II. preced.) It has therefore, when it floats in ſtill water, always, for its point of ſuſpenſion, the point of its capacity which is perpendicularly and immediately above the point of ſection of the vertical of the centre of gravity of its immerged part by the plane of its actual flotation.

And conſequently, its line of ſuſtentation, ſuch as I have determined (§. IV. preced.) is always in the plane of its actual flotation, and its axis of inclination, ſuch as I have determined, (§. V. preced.) is always ſuſtained lengthwiſe on this plane of its actual flotation.

§. XV. To be ſatisfied that its point of ſuſtentation does always reſide in the plane of its actual flotation, whatever the liquids be on which it floats, be they heavy or light, ſtagnant, current, or agitated, and whatever be the forces applied to it; I cauſe it in my imagination to float on liquids of different weights, on ſtill and running waters, which I make it to paſs through; and finally, I commit it to the rage of the winds, and of an agitated ſea, which lift it up ſucceſſively to the top of waves of an immenſe height, and then let it down into deep abyſſes. To every quantity thereof immerged in theſe liquids, to every height, and to every different ſituation which it acquires ſucceſſively, I mentally affix for a ſingle moment, all the impreſſions of the forces which act upon it, that I may have leiſure to apply to the part thereof that is actually immerged, all the experiments and demonſtrations, which I have mentioned in §. XII, XIII, and XIV aforegoing: and I find, that they lead me always to the ſame determination of the continual reſidence of its point of ſuſtentation, in the point of ſection of the vertical of the centre of gravity of the part actually immerged, by the plane of its actual flotation, and to the ſame determination of its point of the ſuſpenſion, of its line of ſuſtentation, and of its axis of inclination.

§. XVI. We have even a confirmation of this truth, in thoſe ſmall enamelled figures, which are kept ſuſpended in water by the means of ſmall empty bubbles above their heads. In the various motions given them by ſhaking the bottle wherein they are contained, it is eaſy to obſerve an oſcillation, whoſe centre of rotation is in the top of the bubbles over their heads: a proof that their point of ſuſtentation is in that top; and which confutes the opinion of thoſe, who ſuppoſe, that a ball plunged in water, in ſuch ſort that the moſt elevated point of its ſurface be on a level with the ſurface of the water, has not its point of ſupport in that point of its ſurface which alone forms its flotation.

Nay farther: Let ſuch bubble be equal in weight to the bulk of water which it diſplaces when it is intirely immerged, and let it be made to ſwim between two waters, provided it be big enough to render ſenſible the different motions impreſſed on it. There will in this caſe be produced an oſcillation, which ſhall have its centre of rotation, in the moſt elevated point of its ſurface. This motion of oſcillation is common to all floating bodies, when the forces which have produced their inclination, do ſuddenly abandon them.

§. XVII. Fix ſucceſſively at different heights in the middle of the ſides of a rectangular concave parallelopiped, two horizontal gudgeons, ſhaped beneath to acute angles, one on the right, and the other on the left ſide, and at every one of the different heights whereat you fix the gudgeons, ballaſt the parallelopiped, ſo as to make it ſink in water, till the lower edges of the gudgeons juſt touch the water's ſurface, and when it is upright, ſhift part of the ballaſt to one end, in order to make it incline lengthwiſe.

1. Provided that its inclination permits its baſe to remain all under water, the gudgeons will, in its different inclinations, continue to be ſupported on the water's ſurface, in the ſame manner as if they reſted on a marble table.

2. If at each different height to which you fix the gudgeons and the quantity of its draught of water, you ſuſpend it in air by placing its pins on two cuſhions of ſteel, and you then ſhift part of the weight, as before; theſe different experiments will make you perfectly ſenſible of the three hypotheſes of the ſhifting of the point of ſuſtentation of any floating body in its inclinations.

Let the half breadth of the ſide, on which it inclines, be $=a$
Its draught of water, when it does not incline $=c$.
The ſine of the angle of its inclination $=x$.

FIRST HYPOTHESIS. The parallelopiped has, even in water, a point of ſuſtentation both before, and after its inclination. And its inclination is the ſame in air and in water, with the ſame ſhifting of the weight, whenever the ſine of its angle of its given inclination $=x$, is $=\sqrt{3c^2-2a^2}$.

Which evidently proves, that its line of ſuſtentation and its axis of inclination are before and after its inclination, the ſame in water and in air: for it has in air for the line of ſuſtentation, that which is terminated by the points of the ſurface of the ſteel gudgeons, on which its iron pins are ſupported, and for the axis of inclination, the line which is terminated by the points of the ſurface of the ſteel cuſhions on which the gudgeons are ſupported. But the diſpoſition and the ſituation of the heavy parts of the parallelopiped, and its inclination being the ſame in air and in water, its line of ſuſtentation, and the axis of its inclination are neceſſarily ſituated in the ſame manner in the two fluids. It has conſequently in water, for its line of ſuſtentation, that which it terminated by the particles of the ſurface of the water, on which the under edges of its gudgeons are ſupported; and for its axis of inclination, that which is terminated by the under edges of the ſaid gudgeons: which is a farther proof of the determination which I have given of the line of ſuſtentation, and the axis of inclination of any floating body.

**Second Hypotheſis.** The point of ſuſtentation of the parallelopiped ſhifts, during its inclination, towards the ſide on which it inclines, and its inclination is leſs in water than in air, with the ſame ſhifting of the ſame weights, whenſoever that,

1. Its given draught of water $= c$, is $= \sqrt{2\,a^2}$.
2. Its given draught of water $= c$ is leſs than $\sqrt{\frac{2\,a^2}{3}}$
3. Its given draught of water $= c$ is greater than $\sqrt{\frac{2\,a^2}{3}}$ and that the ſine of its given angle of inclination $= x$ is greater than $\sqrt{3\,c^2 - 2\,a^2}$.

Becauſe its line of ſuſtentation and axis of inclination remains always in the middle of its length, when it is ſuſpended in air by its gudgeons, whereas when it floats they are diſplaced, by a certain quantity, during its inclination, towards the ſide to which it inclines; that by their being diſplaced, they diminiſh by the ſame quantity the leavers on which thoſe of its heavy parts that are on the ſame ſide, and which have been augmented by the ſhifting of the weights, do act, are diminiſhed by this quantity; and that they increaſe by the ſame quantity the leavers on which thoſe of its heavy parts which are on the oppoſite ſide, and which have been diminiſhed by the ſhifting of the weights: ſo that a leſs inclination in water, is ſufficient to eſtabliſh the equilibrium between the oppoſite ſides of the perpendicular of its point of ſuſtentation.

**Third Hypotheſis.** The point of ſuſtentation of the parallelopiped ſhifts in water towards the ſide oppoſite to that whereon it inclines, and its inclination is greater in water than in air, whenever its given draught of water $= c$, is greater than $\sqrt{\frac{2\,a^2}{c}}$ and the given ſine of the angle of its inclination $= x$, is leſs than $\sqrt{3c^2 - 2a^2}$.

Becauſe that its line of ſuſtentation, and its axis of inclination, do remain always in the midſt of its length, when it is ſuſpended by its gudgeons in air, whereas when it floats, they ſhift, during its inclination, by a certain quantity towards the ſide oppoſite to that whereon it inclines; that by their ſhifting they augment by that quantity the length of the leavers on which thoſe of its parts which are on the ſide on which it inclines, do act, and which have themſelves been augmented by the tranſlation of the weights; and that they do diminiſh, by the ſame quantity, the length of the leavers whereon thoſe of its heavy parts do act, which are on the oppoſite ſide, and which have themſelves been diminiſhed by the tranſlation of the weights; ſo that it muſt in water undergo a greater inclination than in air, to be able to reſtore the equilibrium between the oppoſite ſides of the perpendicular of its point of ſuſtentation.

It is neceſſary to obſerve, with regard to all other floating bodies, that the configuration of their parts, which are ſucceſſively immerged, and the quantity of them that is plunged under water, have an abſolute connexion with the poſition of the centre of gravity of ſuch immerged parts, and the ſhifting of their points of ſuſtentation, which is ever relative to ſome one of the three hypotheſes which I have been explaining.

§. XVIII. All the points of the capacity of a floating body, through which when it is erect, the perpendicular of the centre of gravity of its immerged part paſſes, do during its inclination deſcribe an arc or portion of a curve, which has continually its centre or focus in its point of actual ſuſpenſion. Such of thoſe points as are above the point of ſuſpenſion, deſcend on the ſide on which it inclines, always increaſing more and more their diſtance from the perpendicular of this point of ſuſpenſion. Thoſe which are under it increaſe their diſtance likewiſe from this perpendicular, but do aſcend on the oppoſite ſide to that on which it inclines.

But when it is erect, its centre of gravity is always[a] in one of the points I have been ſpeaking of, without which it would not then be in the perpendicular of the centre of gravity of the immerged part, exactly ſupported by its actual point of ſuſtentation. It remains during the inclination in the ſame point of the capacity of the body where it was before, and being ſubjected to the motion of the point in which it reſides, it moves differently, as it is more or leſs elevated. But by its motion it permits, occaſions, or hinders the overſetting of the floating body in the manner I am going to explain.

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[a] The laſt concluſion of §. I.

1°. In the caſe of the firſt hypotheſis, the floating body overſets upon the leaſt inclination, when being erect, its centre of gravity is in its point of ſuſpenſion; becauſe then this centre of gravity turns on its ſelf, and gives no reſiſtance to the forces which occaſion its inclination.

2°. The floating body overſets upon the leaſt inclination, whenever by the motion which its inclination gives to its centre of gravity, this centre of gravity is tranſlated without the perpendicular of its point of actual ſuſtentation on the ſide on which the body inclines; becauſe from the inſtant it is on that ſide, its motion tends to remove it further and further from the perpendicular of its point of ſuſtentation, and being continually leſs and leſs ſupported by this point of ſuſtentation, it concurs with the forces which occaſion the inclination, to overſet the floating body, which happens:

In the caſe of the firſt hypotheſis, when the floating body being erect, its centre of gravity is more elevated than its point of ſuſpenſion.

In the caſe of the ſecond hypotheſis, when the floating body being erect, its centre of gravity is too much elevated above its point of ſuſpenſion to permit that the quantity whereby this point of ſuſpenſion ſhifts towards the ſide on which the body inclines, ſhould exceed that whereby the centre of gravity moves towards the ſame ſide.

In the caſe of the third hypotheſis, whenever the floating body being erect, its centre of gravity is more elevated than its point of ſuſpenſion, and when being below this point it is too little to permit that the quantity whereby it moves towards the ſide oppoſite to that whereon the body inclines, ſhould exceed that whereby the point of ſuſtentation ſhifts towards the ſame ſide.

3°. The floating body may incline without overſetting, whenever by the motion which its inclination gives to its centre of gravity, this centre of gravity is without the perpendicular of the point of its actual ſuſtentation on the ſide oppoſite to that on which the body inclines; becauſe then this motion tends to remove it further and further from the perpendicular of its point of actual ſuſtentation, and acts then ſo as to counter-balance the forces which occaſion its inclination, on a leaver equal to the diſtance it is at from the perpendicular of its point of ſuſtentation; which happens, In the caſe of the firſt hypotheſis, when the floating body being erect, its centre of gravity is below its point of ſuſpenſion.

In the caſe of the ſecond hypotheſis, whenever the floating body being erect, its centre of gravity is below its point of ſuſpenſion; and when it is above it, but too little elevated for the quantity whereby it moves towards the ſide on which the body inclines, to come up to that whereby its point of ſuſtentation ſhifts towards the ſame ſide.

And in the caſe of the third hypotheſis, when the floating body being erect, its centre of gravity is below, and ſufficiently diſtant from its point of ſuſpenſion, for the quantity whereby it moves towards the ſide oppoſite to that on which it inclines, to exceed that whereby its point of ſuſtentation ſhifts towards the ſame ſide.

Now it is eaſy to apprehend from the foregoing recital,

That the third hypotheſis is very diſadvantageous to it, ſince in the caſe where it takes place, its point of ſuſtentation ſhifts towards the ſide oppoſite to that on which it inclines.

That the firſt would be leſs diſadvantageous to it, if it could take place therein without the neceſſity of firſt paſſing through the third hypotheſis. In effect this firſt hypotheſis is impoſſible for the parallelopiped, when its given draught of water $= c$ is equal to or leſs than $\sqrt{\frac{2a^2}{3}}$. It cannot take place but when its draught of water is greater than $\sqrt{\frac{2a^2}{3}}$: but it is then neceſſary that it ſhould acquire an inclination great enough for the ſine of the given angle of its inclination $= x$ to become $= \sqrt{3c^2 - 2a^2}$ without which its point of ſuſtentation would not be after its inclination, in the point where it was before.

Finally, that the ſecond hypotheſis is the moſt advantageous of all to it, provided care be taken to manage its draught of water relatively to the configuration of its immerged part, and in ſuch ſort that its point of ſuſtentation be forced to ſhift towards the ſide on which it inclines, from the firſt inſtant of its inclination. An advantage aſſuredly appertaining to the floating parallelopiped, taking care to keep its draught of water leſs than the root of two thirds of the ſquare of half its breadth, by means of which the ſhifting of its point of ſuſtentation tends always to augment the length of the leaver on which its centre of gravity acts to oppoſe its inclination, even though this centre of gravity be above the plane of its flotation: a ſenſible example whereof are lighters laden with hay, coals, or wood.

§. XIX. From theſe different inquiries it appears that the floating body cannot properly incline, unleſs its point of ſuſtentation ſhifts. But to prove that the different ſhiftings of that point of ſuſtentation produce no alteration in the mechaniſm of its motions, I imagine it ſuſpended in air by the point of a pyramid inverted, and by the help of an external force, and therein I make it ſucceſſively to run through all the points which it did when ſuſpended in water. And I obſerve, that during the applications of the ſame forces, it obſerves in air the ſame poſitions it before acquired in water; becauſe its point of ſuſpenſion, ſuch as I determine it, being during each of the different poſitions which it acquires and obſerves, always in the perpendicular of the centre of gravity of its immerged part, the centre of gravity common to its heavy parts, and to the forces which are applied to it, is in air, as in water, in that ſame perpendicular, exactly ſuſtained by its point of actual ſuſtentation.

§. XX. It is true this laſt demonſtration is called in queſtion, by alledging, that if during the application of the ſame forces, I apply the point of my inverted pyramid to any other point I pleaſe of the perpendicular of the centre of gravity of the part that is immerged during a determinate poſition, it will acquire the ſame poſition in air, becauſe the centre of gravity common to its heavy parts, and to the forces which are applied to it, will be equally ſupported by its point of actual ſuſtentation.

This objection might have ſome foundation, if all the forces which occaſion or which hinder the inclination of the floating body, had their direction perpendicular to the horizon; becauſe the leavers upon which they would act, being always equal to the diſtances between the point where they apply their centre of effort, and the perpendicular of its point of ſuſtentation, which is that of the centre of gravity of the immerged part, the height of this point of ſuſtentation has no influence upon the length of their leavers. But it is not the ſame with forces whoſe directions are horizontal: the leavers on which they act are always [a], equal to the diſtance between their line of direction and the horizontal plane of the point of ſuſtentation of the floating body. The experiments which I have mentioned, the demonſtrations I have brought, and the computations which I have been able to make in conſequence, do all of them invincibly prove, that forces whoſe directions are horizontal do act in occaſioning or impeding the inclination of the floating body, on leavers always equal to the diſtance between the line of direction of their centre of effort, and the plane of the actual flotation of the floating body; and that ſuppoſing its point of ſuſtentation to be more or leſs elevated than the plane of its actual flotation, it is impoſible to eſtabliſh the equilibrium which ſubſiſts between theſe forces, and thoſe which counterbalance them when the floating body remains fixed in a determinate ſituation.

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[a] The third obſervation of §. VIII.

§. XXI. The mechaniſm which is the object of this eſſay is of too intereſting a nature, not to engage the greateſt geometricians to conſider and explain it, and to communicate to me their ſentiments on this ſketch of my principles. Indeed this mechaniſm, though practiſed at all times, by all nations, is not as yet rightly underſtood, and yet on its laws it is that the goodneſs of veſſels only depends: they alone can furniſh the means to fix their proportions, the configuration of their immerged parts, the diſpoſition and proportions of their maſſs, and their ſtowages, which are abſolutely arbitrary. As long as we ſhall be without ſuch laws, we ſhall never ſee a perfect veſſel, nor be aſſured it has all the advantages which might attend its ſailing.

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# EXPOSITIONS
On the EXTRACT of the
# MECHANISM
Of the MOTIONS of
# FLOATING BODIES.

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## PART I.

SHEWING the certainty and uſefulneſs of my diſcovery of the mechaniſm of floating bodies.

### §. I. *Of the point of ſuſtentation of floating bodies.*

When a body floats at freedom, in ſtill water, all geometricians agree with me,

1. That the efforts of the preſſures of the water againſt the immerged part thereof, are the [a] ſole cauſe of its being ſupported and buoyed up.
2. That they act in a vertical direction.
3. That the whole force thence ariſing is equal to the weight of the diſplaced bulk of water, to that of the floating body, and to the ſupport which ſuch body requires.
4. That their centre of [b] effort is always in the perpendicular of the centre of [c] gravity or magnitude of the immerged part of the floating body.
5. That when ſuch body remains fixed in a determinate poſition, its centre of gravity is in the very ſame perpendicular, exactly ſup-

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[a] §. I. Of my extract.

[b] §. XII. Of my extract.

[c] Note, that by the centre of gravity or magnitude of the immerged part, I mean the point where its centre of gravity would be, if it were homogene in all parts of its capacity.

ported by the centre of effort of the preſſures of the water. This centre of effort of the preſſures of the water muſt therefore in this caſe have a force equal to the whole of that ariſing from their efforts, and to the ſupport which the centre of gravity of the floating body requires; it is therefore in this caſe the point of ſuſtentation of this centre of gravity; and conſequently it is that of the floating body.

But its force, equal to the weight of the bulk of the diſplaced water, to that of the floating body, and to the ſupport which the floating body requires, is aquired at the inſtant the floating body diſplaces a bulk of water whoſe weight is equal to its own. Indeed this force ariſes ſolely from this; that it is the centre of the ſupports of the ſaid body; and is conſtantly determined by the weight of the bulk of the diſplaced water; the poſition of the centre of gravity of the body is no way concerned in it, it being ever the ſame whether the centre of gravity be in the perpendicular of the centre of magnitude of the immerged part, or out of it, more or leſs above or below the ſurface of the water it floats on. The centre of effort of the preſſures of the water is therefore at all times the [a] point of ſuſtentation of the floating body, ſince whatever be the poſition of the body, it is conſtantly the centre of its ſupports, having ever a force equal to the ſupport which its centre of gravity requires.

## §. II. *Of the height of the point of ſuſtentation of the floating body.*

The force of the centre of effort of the preſſures of the water, and the quality of the point of ſuſtentation of the floating body being thus ſtated, and the perpendicular of the centre of magnitude, in which I place it, being [b] allowed by all geometricians [c], there ſeems to be no room to doubt of its conſtant reſidence [d] in the point of the ſection of this perpendicular by the plain of actual flotation of the floating body. In reality the height I have aſſigned [e] to it is no arbitrary determination; but ſtrictly conformable to the laws of motion whereby, when one or more forces act at the ſame time upon the ſame body, their centre of effort is always in the point of ſection of the line of the direction of their common centre by the moſt extreme of the plains which are perpendicular to that direction, and terminated by the parts to which they are applied. To be convinced of the truth of this, when the floating body remains at reſt in a given poſition, I ſuppoſe [a] its gravity to be annihilated, and ſupplied by ſome different force equal to it, and acting in the ſame direction towards the earth's centre being applied to the upper part of the body. Then I imagine the body to be divided into infinitely thin horizontal ſlices, which I ſuppoſe inflexible. I take it out of the water and cauſe it to be ſupported in air in the ſame condition and poſition, by means of ſprings, whoſe vertical direction is parallel to the direction ariſing from the efforts of the water's preſſures, and theſe are applied to all parts of the under ſide of the baſe, and about all the ſlices of which its immerged parts under the circumſtance of floating is compoſed. The elaſtic force which theſe ſprings acquire from their compreſſion, by yielding to the new force which I have applied to the top of the body, is exactly equal to this new force, with which it is in *equilibrio*; to the ſupport which the body requires to reſiſt its impulſe, and conſequently to the whole force ariſing from the efforts of the preſſures of the water: moreover theſe ſprings have the ſame direction as the preſſures of the water, and they are applied to the very ſame parts; ſo that, like them, they have their centre of effort in the perpendicular of the centre of magnitude of the immerged part. In reality, the new force applied to the top of the body has its direction in this perpendicular; and if it were not exactly ſuſtained there by their centre of effort, it would cauſe the body to incline and alter its poſition; contrary to the hypotheſis.

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[a] §. I. Of my extract.

[b] §. XII. Of my extract.

[c] Ibid.

[d] §. XIV. Of my extract.

[e] §. XIII. Of my extract.

Whilſt the body is ſupported by theſe ſprings, I ſieze one of the ſlices of the immerged part by its ſides, and diſengage the top of it from the force which was applied thereto; and this operation I repeat from ſlice to ſlice, from the baſe quite up to the plain of flotation; and I do the like when it is in the circumſtance of floating, and with reſpect to every ſlice which I keep thus apprehended: I obſerve,

1°. That provided it be beneath the plain of flotation, all the ſlices which are above it, quit it, to yield to the efforts of the ſprings, or preſſures of the water which act upon them, which informs me that they communicate to it none of the impulſes which ſuch efforts impreſs upon them.

2°. That if I ſieze that which forms the plain of the body's flotation, the whole body remains at perfect reſt; whence I infer, that the ſlices above the plain of flotation do not receive, either from the ſprings, or the preſſures of the water, any other impulſe than that which the ſlice of the flotation communicates and transfers to them;

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[a] §. XIII. Of my extract.

ſo that the re-union of the efforts of theſe ſprings, or preſſures, is effected before their impreſſions reach the ſlices below the plain of flotation.

3°. That the force which I want to prevent any given ſlice from yeilding to the impulſes of the ſprings, or the preſſures of the water which act upon it, and upon the ſlices under it, is equal to their whole force; and that theſe ſprings, or thoſe preſſures of water, do conſequently, in ſo doing, unite all their efforts.

4°. That the force requiſite to keep the ſlice of flotation in this manner, is equal to the whole of that of the ſprings or preſſures of the water which act upon the whole immerged part; ſo that all their efforts are united at the ſlice of flotation.

5°. That in like manner I prevent any one ſlice from yielding to the impulſes of the ſprings, or the preſſures of the water which act upon it and upon the ſlices beneath it, by applying a force equal to the whole force of ſuch ſprings or ſuch preſſures at the point of ſection of the line of direction of their common centre by that particular ſlice; but that if I apply my force to any other point of the ſurface of ſuch ſlice, it will incline, and by ſliding away under my force, will eſcape together with the reſt of the ſlices beneath it; which gives me to underſtand, that the centre of effort of the ſprings, or the preſſures of the water, which act upon theſe different ſlices, is in the point of ſection which I juſt now aſſigned; and conſequently that the centre of effort of the ſprings or the preſſures of the water, which act upon the immerged part, is as I have determined it [a], in the point of ſection of the vertical of the centre of magnitude of ſuch immerged part, by the plain of flotation; inaſmuch as by applying a force equal to theirs at this particular point of ſection, I keep the whole body in the determinate poſition, whereas were I to act on any other point of its ſuface of flotation, it would incline and eſcape away.

## §. III. *Objections obviated.*

To this determination of the centre of effort of the ſprings or preſſures of the water is objected the uſual manner of conſidering the gravity of a body as united in its centre of gravity; a conſideration juſtified by an infinite number of experiments, and which ſeems to point out the centre of magnitude of the immerged part, for the place of the centre of effort of the preſſures of the water. But in the mean time, it is not taken notice of, that this mode of conſideration can only take place, when the body is applied to a leaver, becauſe in that caſe, if every one of its gravitating particles be multiplied by that leaver whereon it acts, and the ſeveral products be ſummed up, it will be equal to the product of the whole gravity of the body, multiplied by the leaver of its centre of gravity. Beſides, the expreſſion commonly made uſe of on this occaſion is abſolutely improper, becauſe it calls that the leaver of the centre of gravity which is in reality that of the centre of effort. The miſtake ariſes from hence, that theſe two centres have one and the ſame line of direction, and that the leaver of the centre of effort is preciſely that on which the centre of gravity appears to act, it being equal to the diſtance between their common direction and the axis on which the body moves, ſo that the cuſtomary conſideration no way invalidates my determination of the centre of effort. Mine is general, and obtains alike in the caſe propoſed, and all others, whether the body moves towards a given place, or endeavours it, as when it reſts on another body which ſupports it, or endeavours to protrude, or ſtrike ſuch body: In all theſe ſeveral caſes, imagine the body to be divided into infinitely thin ſlices perpendicular to its actual direction, and then treat it according to the ſeveral modes deſcribed in the laſt §. and you will plainly perceive that its centre of effort is ever in the point of ſection of the line of direction of its centre of gravity by the moſt extreme of the ſlices into which you have divided it.

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[a] §. XIV. Of my extract.

Again, it is objected that treating a body according to my directions, is deſtructive of the very nature thereof, and that ſuch diviſion into infinitely thin ſlices perpendicular to its actual direction, furniſhes me with a pretext for confining to one ſingle point, a property which is common to an infinite number of others. Take, for Inſtance, the body which I ſupport alternately in air, by ſprings, and in water, by the preſſures of that fluid, and they will tell you, that ſuppoſing as I have done, its gravity annihilated, and another foreign force ſubſtituted in its ſtead, I may put it in what poſition I pleaſe, by applying ſuch new force to whatſoever point of the perpendicular of the centre of gravity of the immerged part I ſhall think fit; provided I permit it to ſubſiſt entire, without disjoining its parts by my diviſion, and the point to which I apply this new force, be below the plain of flotation; or otherwiſe, dividing it as I do, this point be above the plain of flotation. But theſe two laſt objections have no better foundation than the former. They are calculated to aſcribe effects to the efforts of the ſprings, and the preſſures of the water, which really do not belong to them. I have indeed provided againſt them by my two firſt obſervations in the foregoing §. I ſhall here only add, in juſtification of my procedure, that the horizontal connections of the body propoſed, are indiſpenſably neceſſary for the uniting in the ſame perpendicular the ſeveral efforts which the ſprings, or the preſſures of the water produce upon the length and breadth of the immerged part: but the perpendicular connections are abſolutely needleſs towards producing the union of theſe efforts at a given height. By ſuppreſſing theſe perpendicular connections, as I do when I divide the body propoſed into infinitely thin horizontal ſlices, I have it in my power to get rid of every thing that is foreign to the object of my enquiry, for the inveſtigation of the effects which are produced ſolely by the impreſſions of the ſprings, or the preſſures of the water, and for the determination of their centre of effort. My diviſion, inſtead of interrupting their action, gives it the privilege of acting at freedom. It inſtructs me, that their direction being vertical, ſuch of their efforts, as act upon the ſuperior parts, cannot of their own accord communicate themſelves to the inferior ones, ſo as there to eſtabliſh their common centre of effort; it affords me the power of purſuing them through theſe ſucceſſive impulſes, by means of which thoſe which act upon the inferior ſlices, are tranſmitted intirely from ſlice to ſlice, quite up to that of the flotation, where they all unite; that being the higheſt of all thoſe to which it is applied, all the ſuperior parts are utterly unconcerned therein, and are ſo helpleſs and foreign to this purpoſe, that whether I permit them to ſubſiſt, or intirely ſuppreſs them, or ſubſtitute ſome other body in their ſtead, the union of the efforts of the ſprings, or of the preſſures of the water in the plain of flotation, and the exiſtence of their centre of effort in the point of ſection of the perpendicular of the centre of magnitude of the immerged part by the plain of flotation, will not be at all the leſs real.

## §. IV. *Of the identity of the mechaniſm of the leaſt compounded motions of floating bodies, with thoſe of the little figures which we commonly call equilibres.*

The centre of effort of the preſſures of the water ſhould therefore be regarded as a ſteady, fixed and immoveable point of ſuſtentation [a] of the floating body, ſince it has at all times a force [b] equal to the ſupport which the centre of gravity of ſuch body requires; and that it ever reſides [a] in the point of ſection of the vertical of the centre of magnitude of the immerged part of the floating body by the plain of its actual flotation; without a poſſibility of its being out [b] of that perpendicular, or above or below [c] the plain of flotation.

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[a] §. I. Of my extract.

[b] Ibid.

This truth being eſtabliſhed, let us take into conſideration thoſe ſmall figures of wood or ivory which we commonly call *equilibres*, fixed to inverted pyramids whoſe vertex ſerves them for a pivot; ſo that being placed upon a ſtand, they will aſſume an erect poſition, or any other you pleaſe, by means of a double counterpoiſe which hangs lower than the ſurface of their ſtand. Hitherto they have been looked upon only as children's toys, and conſequently their mechaniſm has been unheeded by the geometricians, yet at this time of day they may be deemed not unworthy of their profoundeſt ſpeculations, upon account of the conformity there is between them, and the motions of floating bodies.

The little figure we are here conſidering, has for its point of ſuſtentation, that point of the ſurface of its ſtand on which the vertex of its inverted pyramid bears; which point meets with, in its point of ſuſtentation, an obſtacle which its tendency to the centre of the earth cannot ſurmount; the centre of gravity of the little figure, and all its appendages, however remote from it, are kept ſuſpended thereby; conſequently this muſt be the point of ſuſpenſion of the figure.

That point of the capacity of a floating body which is immediately above its point of ſuſtentation, ſuch as I have juſt now determined, meets with an obſtacle in that point of ſuſtentation [d] which is likewiſe as inſurmountable to its tendency towards the earth's centre, as is that which the vertex of the inverted pyramid of the little figure meets with in that point of the ſurface at the ſtand on which it bears. The centre of gravity of the floating body, together with all its connected appendages, at whatſoever diſtances from the point immediately above its point of ſuſtentation, are thereby kept ſuſpended; and conſequently that point muſt be the point of ſuſpenſion of the floating body.

The parallel I have been drawing between the points of ſuſtentation and ſuſpenſion of the little figure propoſed, and of the floating body, appears to me alike ſimple and evident. And I may readily conclude from thence, that thoſe two bodies are in their motions ſubject to the very ſame laws [e]. The following reflexions will put this matter beyond all doubt,

When the propoſed little figure inclines [f]:

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[a] §. II. Of my extract.

[b] §. I, and XI. of my extract.

[c] Ibid.

[d] §. II. Of my extract.

[e] §. III. Of my extract.

[f] §. IV. Of my extract.

1°. It has for a line of ſuſtentation, the parallel to the horizon which croſſes the ſurface of the ſtand, which paſſes through its point of ſuſtentation, and which is alſo parallel to the length of the ſide on which it inclines.

2°. It has for an axis of inclination the parallel to the horizon which paſſes through its point of ſuſpenſion, and which bears all along upon its line of ſuſtentation.

3°. Its centre of gravity, and all its other parts, during its inclination, deſcribe arcs or portions of curves, which are parallel to each other, and have always their centre or focus in its axis of inclination, and whoſe ſurfaces are perpendicular to that axis.

And as it is at liberty to incline on every one of its ſides [a] it may be conſidered as having as many poſſible lines of ſuſtentation, and axes of inclination, as we can conceive diameters in the two horizontal circles which would have its points of ſuſtentation and ſuſpenſion for their centres.

When it turns round upon itſelf [b] it has for an axis of ſuch circular motion the perpendicular of its centre of ſuſtentation, and all its parts deſcribe arcs or ſpirals whoſe centre or focus is ever in that perpendicular.

All theſe ſeveral determinations of its lines of ſuſtentation, of the axis on which it moves, and of thoſe arcs or portions of curves which its parts deſcribe during its different motions, are exactly the ſame as I have aſſigned [c] in my extract on the mechaniſm of floating bodies, and they are confirmed by all poſſible experiments.

That this parallel may appear the more evident, place a figure ſimilar to the former on a hollow bowl or diſh: ballaſt the diſh in ſuch ſort that the ballaſting may not ſhift during its various motions; and when it floats at freedom, and without motion, let the poſition of the little figure it ſuſtains be erect: Laſtly, let its centre be level with the ſurface of the water, and the common centre of gravity of itſelf, its ballaſt and little figure below the plain of flotation. Place theſe two little figures, the former on its ſtand, the latter upon ſtill water, and leave them at freedom: then will you perceive them to fall into an oſcillatory motion continually decreaſing, during which they will both preſerve the ſame point of ſuſtentation, and when it ceaſes, they will reſt upright.

Twirl them round and their motion will be uniform.

Laſtly, practiſe upon them the ſeveral operations ſpecified in my extract[a], of the leavers upon which the forces which are applied to floating bodies do act; of the motions which they impreſs thereon; of the augmentations or the diminutions which they produce in their gravity; and you will find in all theſe caſes, that the mechaniſms of their motions are regulated by the very ſame laws.

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[a] §. V. Of my extract.

[b] §. IV, V. and VI. Of my extract.

[c] Ibid.

§. V. *The identity of the mechaniſm of the moſt complicated motions of floating bodies, and of thoſe of the little figures commonly called equilibres.*

Hitherto I have ſuppoſed the floating body at freedom, in ſtill water, and conſtantly preſerving the ſame point of ſuſtentation; and I have drawn a parallel between it and a little figure fixed to a ſolid inverted pyramid placed upon an immoveable ſtand. When it is in this ſtate, I apply different forces thereto, ſome of which put it into various inclinations; others cauſe it to run along the ſurface of its ſtand, and to turn round upon itſelf; others again make its pyramid to lengthen and ſhorten ſucceſſively and move it all manner of ways: laſtly, others raiſe and lower its ſtand alternately, and impreſs variety of motions thereon, by ſhifting it from ſide to ſide. Amidſt all its different poſitions I find, 1°. that the mechaniſm of its motions cannot be interrupted, either by the alterations of the point of ſuſtentation occaſioned by its march over the ſurface of the ſtand, and the particular motions of the ſtand itſelf, or by the motions whereof they mutually participate. 2°. That in conſideration of its pyramids continually ſhifting its place and height, its point of ſuſpenſion is brought ſucceſſively before, behind, and to the ſides, at greater or leſs diſtance from its counterpoiſe, I perceive that the diſtance between this point of ſuſpenſion and its centre of gravity continually varies, but the mechaniſm of its motions remains ever the ſame; ſince its centre of gravity is at no time, nor in any given ſituation, at all the leſs reſtrained to a like determinate diſtance from its point of actual ſuſpenſion, nor leſs conſtrained to deſcribe, in that ſame poſition, a portion of a determinate curve, whoſe focus is ever in ſuch point of actual ſuſpenſion. Nor are any of the reſt of its parts leſs reſtrained to determinate diſtances from the axis of its actual motion, or leſs conſtrained to deſcribe at the ſame time, portions of curves, whoſe foci are ever in the ſame axis, and their ſurfaces perpendicular thereto.

The different poſitions I have been ſuppoſing it to be in, are abſolutely analogous to thoſe of a ſhip, according to the various action of forces upon her, augmenting and diminiſhing her gravity, according as the water on which ſhe floats is more or leſs light, and ſhe is more or leſs immerged therein, and her points of ſuſtentation and ſuſpenſion are more or leſs raiſed. Moreover ſhe cannot incline any way without an alteration of her immerged part; ſo that her plain of flotation, the centre of magnitude of her immerged part, the perpendicular of this centre of magnitude, the point of ſection of this perpendicular by the plain of her actual flotation, and conſequently her points of ſuſtentation and ſuſpenſion, are all varied according to the different quantities of her inclination, and her immerſion in the water. Whether floating[a] on ſtill water, running, or agitated; or again ſtopt in ſome certain place; now driving down a river, or on a lake, or the ſea, now expoſed to the fury of a moſt turbulent ocean, and alternately raiſed to prodigious heights, and let down to the loweſt abyſs; in all this variety of circumſtances, ſhe is conſtrained to obey the different motions which the winds, the waves and the mariners impreſs upon her. But before I ſuffer myſelf to be ſcar'd at this detail, I obſerve that her wake, her lee-way, and the particular motions of the water which eſcapes under her, produce continual alterations in the volumes of water which ſucceſſively ſupport her. But 1°. That ſuch alterations ſhould be put upon a par with the alterations of the point of ſuſtentation of the little figure, and ſo cannot occaſion any interruption in the mechaniſm of her motions. 2°. That it is juſt the ſame thing with regard to the motions of which ſhe participates in common with the water, on which ſhe floats. 3°. That as often as her point of ſuſpenſion ſhifts, the diſtance between ſuch point of ſuſpenſion and her centre of gravity is varied; but that the mechaniſm of her motions continues the ſame, becauſe, that every moment, and in every given poſition, her point of actual ſuſtentation is not a jot the leſs certainly fixed[b], nor the leſs invariably determined; the ſupport which her point of actual ſuſpenſion meets with, in that point of ſuſtentation, is not at all the leſs ſolid, or the leſs real; her centre of gravity is not a bit leſs confined to a determinate diſtance from ſuch point of actual ſuſpenſion, nor leſs obliged to deſcribe in that determinate poſition a portion of a curve alike determined, whoſe focus is ever in that ſame point of actual ſuſpenſion: in a word, all the reſt of her parts are equally and alike ſubject to determinate diſtances from the axis of her actual motion,

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[a] §. VII, VIII, IX, and X. Of my extract.

[a] §. XV. Of my extract.

[b] §. I, and II. Ibid.

nor leſs conſtrained to deſcribe portions of curves ſimilar and parallel to that which her centre of gravity deſcribes; whoſe foci are conſtantly in that axis, and their ſurfaces perpendicular to it. The laws of the mechaniſm of the motions of floating bodies are at all times conſtantly the ſame, although their effects appear to be infinitely varied. The multiplicity of theſe effects has hitherto occaſioned a belief that the theory of ſuch mechaniſm is a thing impracticable: but it is an eaſy matter to reduce them to a very ſmall number of claſſes; and when in each claſs the moſt rigorous circumſtance the floating body can be conceived in, is once ſatisfactorily cleared up, the reſt are of conſequence infallibly provided for.

### §. VI. *A ſummary of my treatiſe, and the uſefulneſs of my diſcovery.*

To explain my ideas the more evidently, I have begun my treatiſe with this of the mechaniſm of the motions of the little figures of which I have ſpoken in the foregoing ſection. I then come to unfold my ſyſtem, the principles, and experiments whereon it is founded: I draw the parallel between the mechaniſm of the motions of my little figures, and that of the motions of floating bodies, and thence I proceed to the means for rendering my diſcovery of uſe in navigation, to which effect I give inſtructions for performing the requiſite practical operations.

1°. To draw the platform of a ſhip from out to out.

2°. To draw the elevation from ſuch platform.

3°. To make a particular menſuration of her immerged part, peculiar to my ſyſtem.

4°. To know her point of ſuſtentation when ſhe is upright, or in any given inclination.

5°. To ſtow her lading in ſuch manner as ſhall moſt increaſe the force of her centre of gravity, to ſoften her motions, and diminiſh her rolling.

6°. To find that point of her capacity where her centre of gravity reſides, without the leaſt knowledge of the diſpoſition of her lading, though ever ſo different from that which I propoſe.

7°. To determine the length of the leaver whereon her centre of gravity ſhall act, when ſhe is moſt of all inclined, and the force of the wind upon her maſt and ſails at that time.

Theſe practical operations and experiments, applied with a careful management to ſome ſhips, in conjunction with the diſcovery I have made of the laws of the diſplacing of their points of ſuſtentation and ſuſpenſion, will put geometricians and mariners in a condition to give to the immerged part of a ſhip the capacity which her intended uſe requires, and the propereſt figure for ſoftening her rolling, her balancing, and her other motions, and to increaſe her power of carrying ſail. They may fix the height of her maſts, and the extent of her ſails, anſwerable to ſuch force, and the proportions of her maſts, yards, cordage and pullies, together with the trim of her ſails, according to the ſtreſs they are to endure, and to give her the moſt advantageous poſition for her tacking with eaſe; to determine the manner of placing her maſts ſo that ſhe ſhall be neither too ſtiff nor too eaſy; and that ſhe ſhall without any aſſiſtance of her rudder (her ſails bearing equally afore and aft) always run that point of the compaſs to which ſhe has been ſet. By means of theſe ſeveral proviſions, they will have every advantage they can wiſh for in her way of ſailing.

The various diviſions which this firſt part contains, does not appear ſufficient for making out the certainty and advantage of the diſcovery I have made about the mechaniſm of the motions of floating bodies. In the ſecond part, I ſhall apply this mechaniſm to a floating right angled parallelopiped, and I ſhall furniſh the key of the equations which are inſerted in my extract, to point out the caſes wherein the three hypotheſes of the diſplacing its point of ſuſtentation, do take place.

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## PART the SECOND.

### *Of the mechaniſm of the motions of floating bodies applied to a floating right angled parallelopiped.*

### §. I. *Preliminary obſervations.*

*In any propoſed floating right angled parallelopiped.*

WHEN the propoſed parallelopiped is erect, I imagine it to be divided into four other right angled and equal parallelopipeds, by two infinitely thin vertical plains mutually interſecting each other in the middle, at right angles, one of which divides it into two equal parts according to its length, and the other into two equal parts according to its breadth.

FIG.

## FIGURE the FIRST.

*Let* BDOS *be one of the two infinitely thin vertical plains.*

And when it is erect, let

HDOP *be its immerged part.*

HP *its flotation.*

FL *its draught of water.*

C *the centre of magnitude of its immerged part* HDOP.

F *its point of ſuſtentation.*

1°. The perpendicular L *a* which divides the plain BDOS into two equal parts breadthwiſe, ſhews the common ſection of the two vertical plains beforementioned, ſince they cut each other in the middle.

2°. It paſſes through the centre of magnitude C, of the immerged part of the propoſed parallelopiped; ſince the homologous parts of its oppoſite ſides are perfectly equal and ſimilar.

3°. Its point of ſection F by the plain of actual flotation HP [a] is conſequently the point of ſuſtentation of the parallelopiped when it is erect.

Let it then have a given inclination.

BD *The ſide on which it is to incline.*

EFH *The angle of its inclination.*

EDOM *Its immerged part.*

EM *Its flotation.*

r R *and R* [b] *different places of the centre of magnitude of its immerged part* EDOM, *in the three different hypotheſes which I am to demonſtrate.*

f F, *and F different places of its point of ſuſtentation in the ſame hypotheſis.*

F *Its point of ſuſtentation when it is erect, and the common ſection of its flotations* HP *and* EM *and of the perpendicular* L *a.*

*Let there be drawn the lines* r f, JF *and R* T *perpendicular to* EM, *the parallels* r a, *and* RA, *and* U *A.*

*And the inclined line* KF, *whoſe angle of incidence* KFH *to the parallel* H *let be equal to that of the given inclination* EFH.

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[a] §. II. Of the foregoing part, and §. XIV. of my extract.

[b] Note. The letters *A F* and *R* relative to the third hypotheſis, are italick, to diſtinguiſh them from thoſe of the firſt hypotheſis, for want of fit types, as they are in the plates.

When the parallelopiped inclines on its ſide B D, all its parts move at the very ſame time, the ſame way, and parallel to the plain B D O S; conſequently,

1°. The centre of magnitude of its immerged part, and its point of ſuſtentation, do always precede, whilſt it inclines, and after its inclination, in the plain B D O S, which never fails to divide it into two perfectly equal and ſimilar parts.

2°. It has ever the ſame centre of magnitude of its immerged part, and the ſame point of ſuſtentation, as this plain: for if in my imagination I divide its immerged part into infinitely thin ſlices, parallel to the plain B D O S, the immerged parts of thoſe ſlices will all of them form, when it is erect, right angled parallelograms equal to the immerged part H D O P of the ſaid plain; and when it inclines, trapeziums equal and ſimilar to the part E D O M of the ſame plain, which will then be immerged. All theſe ſlices will always, as well as itſelf, have the centres of magnitude of their immerged parts at the ſame height, and at the ſame diſtance from the ſide B D, and like it they will always have their points of ſuſtentation in the plain of flotation, at the ſame diſtance from the ſide B D.

So that whatever I ſhall ſay of the motions of the plain B D O S, of the diſplacing of the centre of magnitude of its immerged part, and of that of its point of ſuſtentation, ſhould be underſtood of thoſe of the propoſed parallelopiped.

### §. II. *The three hypotheſes of the diſplacing of the point of ſuſtentation of a floating right angled parallelopiped ſtated.*

After the parallelopiped has begun to incline, it muſt turn upon its point of ſuſtentation F, ſince that point receives from the efforts of the impreſſions of the water, a force[a] which the gravity of the parallelopiped cannot over-power; ſo that by this motion,

1°. F E is tranſlated to F H; F J to F L; and F M to F P, ſince the angles E F H, J F L and M F P are equal.

2°. The centre of magnitude of the part E D O M which is immerged during the given inclination, may be, either in ſome point, as R, of the line E J: or in ſome point, as *r* without the line F J, towards the ſide B D on which the parallelopiped inclines: or in ſome point as *R*, without the ſame line F J, towards the ſide *S O*, oppoſite to the ſide B D on which the parallelopiped inclines: now that one of theſe three ſeveral reſidences of the centre of magnitude of the part E D O M which takes place in the given inclination, depends upon that one of the three hypotheſes I have undertaken to demonſtrate, wherein the propoſed parallelopiped is with reſpect to the locality of its point of ſuſtentation. For the lines *r f*, R F and *R F*, are given perpendicular to E M, the flotation of the parallelopiped in the given inclination. Wherefore they indicate the ſeveral caſes of the placing of the vertical of the centre of magnitude of the part E D O M which is then immerged; and conſequently their points of ſection *f*, F and *F* by the actual flotation E M indicate the ſeveral caſes of the reſidence of the point of ſuſtentation of the parallelopiped: therefore,

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[a] §. I Of the firſt part.

## HYPOTHESIS the FIRST.

The right angled floating parallelopiped has one and the ſame point of ſuſtentation F both before and after its inclination, whenever the centre of gravity of its part E D O M which is immerged during the given inclination, is in ſome certain point as R of the line F J.

## HYPOTHESIS the SECOND.

The point of ſuſtentation of the right-angled floating parallelogram ſhifts during its inclination, towards the ſide B D on which it inclines, and is tranſlated into a certain point as *f*, whenever the centre of gravity of its immerged part E D O M, at the time it inclines, is in ſome point as *r*, without the line F J, and towards the ſide B D.

## HYPOTHESIS the THIRD.

The point of ſuſtentation of the right angled floating parallelopiped, during its inclination ſhifts towards the ſide S O, oppoſite to the ſide B D on which it inclines, and is tranſlated into a certain point as *F*, as often as the centre of magnitude of the part E D O M, which is immerged during the given inclination, is in a certain point as *R*, without the line F J, and towards the ſide S O.

*Note, I ſhall firſt determine by computation, the ſhiftings of the centre of magnitude of the immerged part* E D O M *of the parallelopiped, in the three hypotheſes I have laid down. Which ſhiftings being known, I ſhall determine thoſe of the points of ſuſtentation and ſuſpenſion of the ſaid parallelopiped. But in order to render my ſolutions the clearer and more intelligible*

*telligible to moſt capacities. I ſhall begin with an expoſition of the practical operations which I have followed in my computations; and independently of the general ſolutions which I ſhall give in algebra, I ſhall to each ſubjoin an example in ordinary numbers. It were to be wiſhed that geometricians, who apply their enquiries to the mechaniſm of the motions of floating bodies, would give themſelves the like pains in behalf of thoſe who are to put their diſcoveries in practice.*

§. III. *Practical operations followed in the computations of the place of the centre of magnitude of the immerged part of a right angled floaing parallelopiped, in its various inclinations.*

*The parallel* H P *ſerving for a fixed line.*

The parallelogram H D O P has its ſuface equal to the product of its breadth H P multiplied by its height F L; its power equal to half its height F L; and its moment equal to the product of its ſurface multiplied by its power.

The triangle E F H has its ſurface equal to the product of its baſe F H multiplied by half its height H E; its power equal to one third of its height H E; and its moment equal to the product of its ſurface multiplied by its power.

The triangles E F H, H F K, and P F M have their ſurfaces, their powers, and their moments equal; ſince their angles, and their ſides are ſimilar and equal; and their baſes in the fixed line H P.

The trapezium E D O M has its ſurface equal to that of the parallelogram H D O P; ſince the triangle E F H to be taken from it, has its ſurface equal to that of the triangle P F M to be added to it for forming the parallelogram H D O P; and to get the moment of the ſaid trapezium we muſt ſubſtract thoſe of the triangles P F M and H F K from that of the parallelogram HDOP; becauſe the triangle P F M makes no part of this trapezium, and the triangle H F K has its moment deſtroyed by that of the triangle H F E oppoſite to it on the other ſide of the fixed line H P, whoſe moment is equal to its own.

The diſtance between the parallel H P and the centre of magnitude of the trapezium EDOM, which is immerged during the given inclination, is equal to the power of that trapezium, and conſequently to the quotient of the moment of the ſaid trapezium divided by its ſurface.

*The perpendicular* L *a ſerving for a fixed line.*

The triangle E F K has its ſurface equal to the product of its baſe E K multiplied by half its height F H; its power equal to two thirds of its height F H; and its moment equal to the product of its ſurface multiplied by its power.

The trapezium E D O M has its moment equal to that of the triangle E F K; becauſe the trapeziums F L D K and F L O M are equal and ſimilar; having their ſimilar ſides and angles equal. And they muſt likewiſe have their reſpective moments equal, having their common ſide in the fixed line L*a*. And finally that being on the oppoſite ſides of this fixed line, their reſpective moments conſequently deſtroy each other, and ſo are to be eſtimated $= 0$.

The diſtance between the perpendicular L *a*, and the centre of magnitue of the trapeizum E D O M, which is immerged during the given inclination, is equal to the power of this trapezium, and conſequently to the quotient of the moment of the ſame trapezium divided by its ſurface.

## §. IV. *A determination of the caſes wherein the firſt hypotheſis of the ſhifting of the point of ſuſtentation of a floating right-angled parallelopiped, takes place.*

### PROPOSITION.

The floating right-angled parallelopiped has the ſame point of ſuſtentation F, before and after its inclination, whenever the height H E of its part E F H, which is immerged during its inclination, is equal to the root of the number which remains after ſubſtracting twice the ſquare of half the breadth F H of the ſide on which it inclines, from three times the ſquare of its water draught F L.

*Or, which is the ſame thing, if we ſuppoſe,*

$a =$ *half the breadth* F H *of the ſide on which it inclines.*
$c =$ *its draught of water* F L *when it is erect.*
$x =$ *the ſine* E H *of the angle* E F H *of its inclination.*

It has the ſame point[a] of ſuſtentation before and after its inclination, whenever the ſine of its given inclination $= x$ is $= \sqrt{3c^2 - 2a^2}$.

### SOLUTION.

The diſtance A F between the parallel H P and the centre of magnitude R of the part E D O M which is immerged during the given inclination, is $= \frac{3c^2 - x_2}{6c}$.

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[a] §. XVII. Of my extract.

To ſatisfy this firſt hypotheſis, the centre of magnitude R of the part EDOM muſt be [a] in the line F J: but the angle J F L is equal to the angle EFH, ſince F J is perpendicular to EM, and FL is perpendicular to H P. The right angled triangles HFE and AFR are therefore equal; ſo then $a =$ the ſide F H of the triangle H F E: $x =$ its ſide E H :: $\frac{3c^2-x^2}{6c} =$ the ſide A F of the triangle A F R: $\frac{2ax}{6c} =$ its ſide AR. Therefore, $\frac{2a^2x}{c}$ the product of F H × AR $= \frac{3c^2x-x^3}{6c}$ the product of E H × A F. So multiplying all by $6c$, and dividing by $x$, I have $2a^2 = 3c^2 - x^2$; and by tranſpoſition, $x^2 = 3c^2 - 2a^2$: which gives me $x = \text{EH} = \sqrt{3c^2 - 2a^2}$, as I ſaid before §. XVII. of my extract. Therefore in the caſe propoſed, the centre of magnitude of the part of the propoſed parallelopiped which is immerged during its given inclination, is in the line F J: and conſequently, the parallelopiped has the ſame point of ſuſtentation before and after its inclination. Which was to be demonſtrated.

## EXAMPLE.

*Let* 2400 = *to half the breadth* F H, *of the ſide on which the propoſed parallelopiped inclines,*
2000 *equal to its draugh of water* F L *when it is erect.*

To have the height H E of the part EFH which is immerged during the given inclination, conformable to the determination I have made by my propoſition.

1°. I multiply the draught of water F L by itſelf, and the product by 3, which makes 12000000.

2°. I multiply the half breadth F H by itſelf, and the product by 2, which makes 11520000.

3°. From the firſt product 12000000
I ſubſtract the ſecond 11520000
The remainder is 480000

4°. I extract the ſquare root of this remainder, which gives me the height required $\text{H E} = 692\frac{820323}{1000000}$.

## SOLUTION.

The diſtance A F between the parallel H P and the centre of magnitude R is $= 960$; and the diſtance A R between the perpendicular L *a* and the centre of magnitude R is $= 277 \frac{1281292}{100000000}$. Therefore $2400 =$ the ſide F H of the triangle E F H : $692 \frac{8203230}{10000000} =$ its ſide H E : : $960 =$ the ſide A F of the triangle A F R : $277 \frac{1281292}{10000000}$ $=$ its ſide A R; therefore $\frac{2400+2771281292}{10000000}$, the product of F H × A R is $= \frac{960+6928203230}{10000000}$ the product of A F × H E. Therefore &c. which was to be demonſtrated.

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[a] §. II. foregoing.

§. V. *A determination of the caſes wherein the ſecond hypotheſis of the diſplacing of the point of ſuſtentation of a floating right angled parallelopiped, takes place.*

## PROPOSITION I.

The point of ſuſtentation of the floating right angled parallelopiped ſhifts, whenever it inclines, towards the ſide BD on which it inclines, and is tranſlated into ſome certain point, as f, whenever F L, its water draught when it is erect, is equal to the root of two thirds of the ſquare of the half breadth F H of the ſide whereon it inclines.

*Or, which is the ſame thing, if we ſuppoſe, as in the foregoing article,* F H $= a$; F L $= c$; *and* E H $= x$.

Its point of ſuſtentation ſhifts[a] during its inclination towards the ſide B D whereon it inclines, as often as its given water draught F L $= c$ is $= \sqrt{\frac{2a^2}{3}}$

## SOLUTION.

The diſtance a F between the parallel H P and the centre of magnitude r of the part E D O M which is immerged during the given inclination is $= \frac{2a^2 - x^2}{6c}$, and the diſtance a r between the perpendicular L *a* and the centre of magnitude r is $= \frac{2a^2x}{6ac}$.

To ſatisfy this ſecond hypotheſis, the centre of magnitude of the part E D O M muſt be in ſome certain point, as r, without [a] the line F J, towards the ſide B D on which the parallelopiped inclines; it is neceſſary therefore that parallel a r which expreſſes the diſtance between the centre of magnitude r, and the perpendicular L *a*, ſhould project without the line F J: but the triangles H F E and a F Z are ſimilar; therefore a = ſide F H of the triangle H F E : $x$ = its ſide H E : : $\frac{2a^2-x^2}{6c}$ = the ſide a F of the triangle a F Z : $\frac{2a^2x-x^3}{6ac}$ = its ſide Z. Whereas the diſtance a r is = $\frac{2a^2x}{6ac}$; it is therefore greater than the ſegment a Z by a quantity = $\frac{x^3}{6ac}$ =the ſegment Z r, whereby the parallel a r projects without the line F J. Therefore in the caſe propoſed, the centre of magnitude of the immerged part of the floating right-angled parallelopiped, and conſequently its point of ſuſtentation, ſhifts towards the ſide B D on which it inclines. Which was to be demonſtrated.

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[a] §. XVII. Of my extract.

## E X A M P L E.

*Let* 2400 *be* = *its half breadth* F H.
1 = *to* H E *the ſine of the angle* H F E *of its inclination.*

I multiply the half breadth F H by itſelf, and the product by 2, and the laſt product comes out 11520000: this I divide by 3 and extract the ſquare root of the quotient 384000, which gives me the height F L of its water-draught, ſuch as I have ſtated in my propoſition, = $1959\frac{591794}{100000}$.

The diſtance a F between the parallel H P and the centre of magnitude r is = $\frac{9215999200000}{94060406112}$ and the diſtance a r between the perpendicular L *a* and the centre of magnitude r is = $\frac{921600000000}{2257449746688}$. Therefore : : 2400 = the ſide F H of the triangle H F E : 1 = the ſide H E : : $\frac{92159992000000}{94060406112}$ = to its ſide a F of the triangle a F Z : $\frac{921599920000}{2257449746688}$ = to its ſide a Z; inſtead of the diſtance a r being $\frac{921600000000}{2257449746688}$ it is therefore greater than the ſegment a Z, by a quantity = $\frac{80000}{2257449746688}$ the ſegment Z r, whereby the parallel a r projects without the line F J;

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[a] §. II. foregoing.

neverthelefs the inclination propoſed in the example is extremely ſmall. Therefore &c. which was to be demonſtrated.

## PROPOSITION II.

The point of ſuſtentation of the floating right angled parallelopiped ſhifts during its inclination, towards the ſide BD on which it inclines, and is tranſlated into ſome certain point as f, whenever its water-draught FL is leſs than the root of two thirds of the ſquare of the half breadth FH, of the ſide on which it inclines.

*Or, which is the ſame thing, if we ſuppoſe, as in the foregoing article,* $FH = a$; $FL = c$; and $HE = x$.

Its point of ſuſtentation ſhifts [a] during its inclination towards the ſide BD on which it inclines, whenever its given water draught $FL = c$ is leſs than $\sqrt{\frac{2a^2}{3}}$.

## SOLUTION.

*Let* $f = \sqrt{\frac{2a^2}{3}}$

$n =$ *to an indeterminate magnitude.*

$c =$ *to its draught of water* $FL = \sqrt{\frac{2a}{3}} - n = f - n$.

The diſtance a F between the parallel H P and the centre of magnitude r, being $= \frac{3c^2 - x^2}{6c}$, is $= \frac{3f^2 - 6fn + 3n^2 - x^2}{6f - 6n.} = \frac{2a^2 - 6fn + 3n^2 - x^2}{6f - 6n}$ and the diſtance $ar$ between the perpendicular L $a$ and the centre of magnitude $r$, is $= \frac{2ax}{6c} = \frac{2ax^2}{6af - 6an}$. Therefore $a =$ the ſide FH of the triangle HFE : $x =$ its ſide EH : : $\frac{2a^2 - 6fn + 3n^2 - x^2}{6f - 6n} =$ the ſide a F of the triangle a F Z : $\frac{2a^2 - 6fnx + 3n^2x - x}{6af - 6anx} =$ to its ſide a Z, whereas the diſtance a r is $= \frac{2a^2x}{6af - 6an}$. Therefore it is greater than the ſegment a Z by a quantity $= \frac{6fnx + x^3 - n^2x}{6af - 6an} =$ the ſegment Z r whereby the parallel a r projects without the line F J. This quantity is in effect a *maximum* ſince the magnitude $f$ is greater than

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[a] §. XVII. Of my extract.

$n$: if they were only equal, the given water-draught F L $= f - n$ would be $= 0$, and the hypotheſis would not obtain. Therefore &c. which was to be demonſtrated.

## EXAMPLE.

*Let* 2400 *be* = *a its half breadth* F H.
1 = *the ſine* H E *of the angle* H F E *of its inclination.*
1959 = *to its water-draught* F L.

The diſtance a F between the parallel H P and the centre of magnitude r is $= \frac{92104336}{9403}$, and the diſtance a r between the perpendicular L $a$ and the centre of magnitude r, is $\frac{92160000}{225676800}$, therefore 2400 = the ſide F H of the triangle H F E : 1 = its ſide E H : $\frac{92104336}{94032}$ = the ſide a F of the triangle a F Z : $\frac{92104336}{225676800}$ = its ſide a Z. Whereas the diſtance a r is $= \frac{921600000}{225676800}$. It is therefore greater than the ſegment a Z by a quantity $= \frac{55664}{2256768800}$ = the ſegment Z r whereby the parallel a r projects without the line F J; however the quantity $\frac{591794}{1000000}$ whereby its water-draught F L is in this example leſs than the root of two thirds of the ſquare of its half breadth F H, and the inclination I have ſuppoſed it to have, are extremely ſmall. Therefore &c. which was to be demonſtrated.

## PROPOSITION III.

The point of ſuſtentation of the parallelopiped ſhifts during its inclination towards the ſide B D on which it inclines, and is tranſlated into ſome certain point, as f, whenever its water-draught F L being greater than the root of two thirds of the ſquare of its half breadth F H, the height E H of the part E F H, which is immerged during its inclination, is greater than the root of the number left after ſubſtracting the ſquares of the half breadth F H, from three times the ſquare of its draught of water F L.

*Or, which is the ſame, if we ſuppoſe, as in the foregoing propoſitions,* F H $= a$; F L $= c$; and E H $= x$.

Its point of ſuſtentation ſhifts [a] during its inclination, towards the ſide B D on which it inclines, whenever its given water-draught F L $= c$, is greater than $\sqrt{\frac{2a^2}{3}}$, and the ſine H E of its given inclination $= x$, is greater than $\sqrt{3c^2 - 2a^2}$

## S O L U T I O N.

*Let* d *be* $= \sqrt{3c^2 - 2a^2}$
$y =$ *an infinitely ſmall magnitude.*
$x =$ *ſine* H E *of the angle* E F H *of its inclination* $= \sqrt{3c^2 - 2a^2} + y$
$= d + y$

The diſtance a F between the parallel H P and the centre of magnitude r, is $= \frac{3c^2 - d^2 - 2dy - y^2}{6c}$ and the diſtance a r between the perpendicular L *a*, and the centre of magnitude r, is $= \frac{2a^2d + 2a^2y}{6ac}$

Therefore [b] $a =$ the ſide F H of the triangle H F E : $d + y =$ its ſide E H : : $\frac{3c^2 - d^2 - 2dy - y^2}{6c} =$ the ſide a F of the triangle a F Z : $\frac{3c^2d + 3cy^2 - d^3 - 3dy^2 - 3dyy - y^3}{6ac} =$ to its ſide a Z. But $d$ is given $= \sqrt{3c^2 - 2a^2}$; $d^2$ is therefore $= 3c^2 = 2a^2$; and ſubſtituting this value of $d$ in that of the ſegment a Z, I have it $= \frac{2a^2d + 6a^2y - 3dyy - y^3}{6ac}$; whereas the diſtance a r is $= \frac{2a^2d + 2a^2y}{6ac}$

It is therefore greater than the ſegment a Z by a quantity $\frac{6c^2y + 3dyy + y^3 - 4a^2y}{6ac} =$ the ſegment Z r whereby the parallel a r projects without the line F J. In effect this quantity is a *maximum*, for the magnitude $6c^2y$, is alone greater than $4a^2y$; ſince $d$, which is given $= \sqrt{3c^2 - 2a^2}$ muſt, according to the hypotheſis, have a conſtant value equal to the root of the number left after ſubſtracting $2a^2$ from $3c^2$. Therefore &c. which was to be demonſtrated.

## E X A M P L E.

*Let* 2400 *be* = *its half breadth* H E.
2000 = *its water-draught* F L.
693 = *the ſine* H E *of the angle* E F H *of its inclination.*

The diſtance a F between the parallel H P and the centre of magnitude r, is $= \frac{92158008}{96000}$; and the diſtance a r between the perpendicular L *a* and the centre of magnitude r, is $= \frac{6386688000}{230400000}$. Therefore 2400 = the ſide F H of the triangle E F H : 693 = its ſide E H : : $\frac{92158008}{9600}$ = the ſide a F of the triangle a F Z : $\frac{63865499544}{230400000}$ = its ſide a Z. Whereas the diſtance a r is $= \frac{63866880000}{230400000}$; it is therefore greater than the ſegment a Z, by a quantity $= \frac{1380456}{230400000}$ the ſegment Z r, whereby the parallel a r projects without the line F J. Yet this is an extremely ſmall magnitude, in compariſon of $\frac{179677}{100000}$, whereby I have ſuppoſed the ſine H E of the angle of inclination, greater than $692\frac{820323}{1000000}$ the root of the number left after ſubſtracting twice the ſquare of half the breadth F H, from three times the ſquare of the draught of water F L, in compariſon of the number two multiplied by the ſquare of the half breadth F H. Therefore, &c. which was to be demonſtrated.

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[a] §. XVII. Of my extract.

[b] Solution of propoſition I. of this ſection.

### §. VI. *A determination of the caſes wherein the third hypotheſis of the ſhifting of the point of ſuſtentation of a rectangular floating parallelopiped, takes place.*

## PROPOSITION.

The point of ſuſtentation of the rectangular floating parallelopiped ſhifts during its inclination towards the ſide O S, oppoſite to the ſide B D on which it inclines, and is tranſlated into ſome certain point as *F*, whenever its water-draught F L, being greater than the root of two thirds of the ſquare of its half breadth F H, the height H E or the part E F H which immerged during its inclination, is leſs than the root of the number which is left after ſubſtracting twice the ſquare of the half breadth F H of the ſide on which it inclines, from three times the ſquare of its draught of water F L, above the number two, multipled by the ſquare of the half breadth F H of the ſide on which it inclines.

*Or which is the ſame thing, if we ſuppoſe as in the foregoing propoſitions* F H = *a*; F L = *c*; *and* H E = *x*.

Its point of ſuſtentation [a] ſhifts towards the ſide O S, oppoſite to the ſide B D on which it inclines, whenever its draught of water F L given $= c$, is greater than $\sqrt{\frac{2a^2}{3}}$, and the ſine H E of the angle EF H of its inclination given $= x$, is leſs than $\sqrt{3c^2 - 2a^2}$.

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[a] §. XVII. Of my extract.

## SOLUTION.

*Let* d $= \sqrt{3c^2 - 2a^2}$
$y =$ *to an infinitely ſmall magnitude*
$z =$ *ſine* H E *of the angle of inclination* E F H $= \sqrt{3c^2 - 2a^2 - y}$
$= d - y$

The diſtance *A* F between the parallel H P and the centre of magnitude *R* of the part EDOM, is $= \frac{3c^2 - d^2 + 2dy - y^2}{6c}$, and the diſtance *AR* between the perpendicular L *a*, and the centre of magnitude *R*, is $= \frac{3a^2 - d - 2a^2y}{6ac}$.

To ſatisfy this third hypotheſis, it is neceſſary that the centre of magnitude of the part EDOM which is immerged during the given inclination, ſhould be in ſome certain point, as *R*, without [b] the line F J, and towards the ſide S O, which is oppoſite to the ſide B D, on which the parallelopiped inclines. But the right angled triangles H F E and *A* F U are ſimilar, ſince the angles at their common point F are equal: therefore $a =$ to the ſide F H of the triangle H F E: $d - y =$ to its ſide HE : : $\frac{3c^2 - d^2 + 2dy - y^2}{6c} =$ to the ſide *A*F of the triangle *A* F U: $\frac{3c^2d - d^3 + 3d^2y - 3dyy - 3c^2y + y}{6ac} =$ to its ſide *A* U. But *d* is given = to $\sqrt{3c^2 - 2a^2}$; therefore $d^2$ is = to $3c - 2a^2$, and ſubſtituting this value of $d^2$ in that of the ſide *A* U, I have it $= \frac{2a^2d + 6c^2y - 3dyy - 6a^2y + y^3}{6ac}$ whereas the diſtance *A R* is $= \frac{2a2d - 2a^2y}{6ca}$; the ſide *A* U of the triangle *A* F U, is therefore greater than the diſtance *A R* by a quantity $= \frac{6c^2y - 3dyy - 4a^2y + y^3}{6ac}$ = the ſegment *R* U, whoſe ſide *A*U projects without the centre of magnitude *R*. Indeed this quantity is a *maximum*: to prove which I firſt of all divide by *y* all the magnitudes which compoſe it; which without interfering with their proportions, gives it $= \frac{6c^2 - 3dy - 4a^2 + y^2}{6ac}$

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[b] §. II. Of my extract.

conſequently I obſerve that $d$ is greater than $y$; for were they barely equal, then x given $= d - y$ would be $= 0$, the inclination of the parallelopiped would amount to nothing, and the hypotheſis would not take place. In conſequence of this obſervation I ſuppoſe $y = d - z$; and ſubſtituting this value in the room of $y$, I have the quantity $\frac{6c^2 - 3dy - 4a^2 + y}{6ac} = \frac{6c^2 - 4a^2 - 2d^2 + dz + z^2}{6ac}$. I cauſe the three firſt terms of this magnitude to vaniſh, by ſuſtituting $3c^2 - 2a^2$ in the room of $d^2$, and I have it $= dz + z'$, which proves it evidently to be a *maximum*. Therefore in the caſe propoſed, the centre of magnitude of the part EDOM which is immerged during the given inclination, and conſequently the point of ſuſtentation of the propoſed parallelopiped do ſhift during its inclination, towards the ſide O S oppoſite to the ſide B D whereon it inclines, which was to be demonſtrated.

## EXAMPLE.

*Let* 2400 = *the half breadth* H F.
2000 = *the water-draught* F L.
692 = *ſine of the angle* E F H *of its inclination.*

The diſtance $A$F between the parallel H P and the centre of magnitude $R$ is $= \frac{92169088}{9600}$ and the diſtance $AR$ between the perpendicular L $a$, and the centre of magnitude $R$ is $= \frac{63774720000}{230400000}$. Therefore 2400 = the ſide F H of the triangle E F H : 692 = its ſide E H : : $\frac{92169088}{9600}$ = the ſide $A$F of the triangle $A$F U : $\frac{6378100889 6}{2304000}$ = to its ſide $A$U; whereas the diſtance $AR$ is no more than $\frac{63774720000}{230400000}$ : this diſtance $AR$ is therefore leſs than the ſide $A$U of the triangle $A$F U by a quantity $= \frac{628896}{2304000}$ = the ſegment whereby this ſide $A$U projects beyond the centre of magnitude $R$. This however is a quantity extremely leſs than $\frac{820323}{1000000}$ whereby I have ſuppoſed the ſine H E of the angle of inclination E F H to be leſs than the root $692\frac{820323}{1000000}$ of the exceſs of three times the ſquare of the draught of water F L above twice the ſquare of the half breadth F H of the ſide whereon the parallelopiped inclines. Therefore &c. which was to be demonſtrated.

§. VI. *A*

### §. VII. *A determination of the ſhifting of the points of ſuſtentation and ſuſpenſion of a rectangular floating parallelopiped, during its inclinations.*

Note. *The angle of inclination of a paralelopiped, and of all other floating bodies, may be known by means of a graduated ſemicircle, or by meaſuring the height* E H *of the part* E F H *which is immerged during its inclination, and the half breadth* F H *of the ſide on which it inclines.*

The ſhifting of its point of ſuſtentation does not take place but in the caſes of the two laſt hypotheſes.

With reſpect to the ſecond hypotheſis.

1°. The ſegment F Q of the parallel H P, and the ſegment r Z of the parallel a r, are equal, ſince they are parallels and are terminated by two parallels f r and F Z.

2°. The ſegment F f of the flotation E M, expreſſes the diſtance between the point of ſuſtentation F of the parallelopiped, when erect, and its point of ſuſtentation f when it inclines.

3°. The triangle f F Q is a right-angled one, and the angle at its point F is that of the inclination of the parallelopiped. Therefore, radius, or ſine of the right angle F f Q : ſine of the angle F Q f, the complement of that of the inclination of the parallelopiped EFH; ; the ſegment F Q = the known one r Z : the diſtance F f between the point of ſuſtentation F of the parallelopiped when it is erect, and its point of ſuſtentation f when it inclines.

This analogy appertains likewiſe to the third hypotheſis, ſince the right-angled triangle F T *F* is ſimilar to F f Q and its ſide *F* T is equal to the ſegment U *R* of the parallel *A R*.

As for what remains, the point of ſuſtentation, and the point of ſuſpenſion of a floating body, have properties ſo different, that it was neceſſary I ſhould fix the diſtinction between them, the better to explain the mechaniſm of the motions of floating bodies. But as they are two infinitely ſmall points, purely metaphyſical, and immediately applied to each other; in computing their height, they ſhould be conſidered as united in one; and in the ſame way the line of ſuſtentation and the axis of inclination of the floating body ſhould be underſtood, and conſidered as united in one line parallel to the horizon.

### §. VIII. *Obſervations on the three hypotheſes of the ſhifting of the point of ſuſtentation of a floating right-angled parallelopiped*

Since the firſt hypotheſis cannot take place, but where the height E H of the part E F H which is immerged during the inclination of the parallelopiped, is equal to the root of the exceſs of three times the ſquare of its draught of water F L, above twice the ſquare of the half breadth F H of the ſide on which it inclines. It follows, in order to ſatisfy this firſt hypotheſis,

1°. That the draught of water F L of the parallelopiped ſhould exceed the root of two thirds of the ſquare of its half breadth: if it be equal [a] to ſuch root, or if it be leſs than it [b], the parallelopiped falls within the caſe of the ſecond hypotheſis,

2°. That in all quantities whereby its draught of water exceeds ſuch root, there is no more than one angle of inclination which can ſatisfy the firſt hypotheſis. If this angle ſhould happen to be too great [c], the parallelopiped is again in the caſe of the ſecond hypotheſis: if it be too ſmall [d], the parallelopiped is in the caſe of the third hypotheſis.

3°. That when the length of the parallelopiped is greater than its breadth, and its draught of water F L is of the ſame magnitude, or leſs than the root of two thirds of the ſquare of its half breadth, it is always in the caſe of the ſecond hypotheſis from the firſt moment of its inclination; becauſe its breadth is the narroweſt of all the ſides on which it can poſſibly incline. But when its draught of water F L is equal to the root of two thirds of the ſquare of its half length, and it inclines on one of its ſides, it paſſes ſucceſſively through all the three hypotheſis; firſt through the third, then the firſt, and laſtly, the ſecond, provided that its inclination be ſufficiently great. So that it is by its breadth alone that its draugth of water ſhould be regulated, to ſecure to it the advantages of the ſecond hypotheſis. I ſhall now ſhew how great thoſe are.

## *Explanation of the ſecond figure.*

### FIGURE II.

*When the parallelopiped is erect, let* B D O S *be the poſition of the vertical plane, cited, for inſtance, in the foregoing ſection.*

H P *its flotation.*

F L *its draught of water.*

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[a] Prop. I. of §. V. of my extract. [b] Prop. II. Ibid.

[c] Prop. III. Ibid. [d] §. IV. Ibid.

HDOP *its immerged part.*
F *its point of ſuſtentation.*
L *a the perpendicular of its point of ſuſtentation* F.
T, J, X, F, Y, K, *and* G *different places of the centre of gravity of the parallelopiped.*

*When it is inclined, let*

b d o s *repreſent its poſition.*
e m *its flotation.*
e d o m *its immerged part.*
r, R *and* R *different places of the centre of magnitude of the immerged part* e d o m, *in the three hypotheſes which I have demonſtrated.*
f, F, *and* F *different places of its point of ſuſtentation in the ſame hypotheſes.*
r f, R F, *and* R F *the perpendiculars of the centre of magnitude of its immerged part, and of its point of ſuſtentation, in thoſe hypotheſes.*
t, i, x, F y k *and* g, *different places of the centre of gravity of the parallelopiped.*

As ſoon as it begins to incline, it turns [a] upon its point of ſuſtentation F, and I conſider [b] then, that all the points of its capacity, which when it is erect, are pervaded by the perpendicular L *a*, deſcribe during its inclination arcs or portions of curves, which have conſtantly their centre or focus in its point of ſuſtentation. The point F has its rotation on itſelf; thoſe which are in F *a*, above the flotation H P approach towards the ſaid flotation, and recede from the perpendicular L *a*, by moving towards the ſide B D on which the parallelopiped inclines. Thoſe which are in F L, below the flotation H P, approach likewiſe towards the flotation, and recede from the perdendicular L *a*, by moving towards the ſide S O, oppoſite to the ſide B D on which the parallelopiped inclines. But when the parallelopiped is erect, its centre of garvity [b] is neceſſarily in the perpendicular L *a*, perfectly ſupported by its point of actual ſuſtentation. And as it remains during the inclination, in the ſame point of the capacity of the parallelopiped, wherein it was before; it is then ſubject to the motions of the point in which it reſides, and its motions are various, as it is more or leſs elevated: but by its motion it permits, produces, or prevents the over-ſetting of the parallelopiped.

It permits it when it is in the perpendicular of the point of actual ſuſtentation; becauſe it then turns upon itſelf, and does not act on any leaver to counterbalance the forces which produce the inclination: which happens in the caſe of the firſt hypotheſis when it is in F; in that of the ſecond, when being at the height of J, it arrives at i during the inclination; and in that of the third, when being at the height K, it arrives at *k*.

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[a] §. XVIII. Of my extract.

[b] §. I. Of the firſt part of this memoir and of my extract.

It occaſions the overſetting of the parallelogram, when it is without the perpendicular of the point of actual ſuſtentation, and on the ſide B D on which the parallelopiped inclines; becauſe in proportion as it moves towards that ſide, it is continually leſs and leſs ſupported by the point of actual ſuſtentation: which takes place, 1°. in the caſe of the firſt hypotheſis, when it is in ſome one of the points X, J, or T, which are more elevated than the flotation H P. 2°. In the caſe of the ſecond hypotheſis, when it is in the point T which is ſo ſufficiently elevated above the flotation, as by its tranſlation to t, towards the ſide B D, on which the parallelopiped inclines, it may exceed the ſhifting F f of the point of ſuſtentation towards the ſame ſide. 3°. In the caſe of the third hypotheſis, not only when it is in the point F of the flotation H P or above that flotation, but when it is below it in ſome certain point, as Y, which is too near it, to be able to reach the ſhifting F *F* of the point of ſuſtentation towards the ſide S O oppoſite to the ſide B D on which the parallelopiped inclines, by its tranſlation to *y*, towards the ſame ſide.

Laſtly, it oppoſes the inclination and the overſetting of the parallelopiped, whenever it's without the perpendicular of the point of actual ſuſtentation, towards the ſide S O, oppoſite to the ſide B D on which the parallelopiped inclines; becauſe it then endeavours to counterbalance the forces which produce the inclination, and the leaver on which it acts for this effect, is equal to the diſtance of it from this perpendicular. So that in the caſe of the firſt hypotheſis, its leaver is equal to the parallel *y* a, when it is in Y; to *k* a, when it is in K, and to g A when it is in G. In the caſe of the ſecond hypotheſis its leaver is equal to the parallel X *x*, when it is in X; to F f when it is in F; to Y Z when it is in Y; to K Z when it is in K; and to G Z when it is in G. In the caſe of the third hypotheſis, its leaver is equal to the ſegment g U when it is in G.

The leaver on which the centre of gravity acts to oppoſe the inclination of the parallelopiped, is therefore in the caſe of the ſecond hypotheſis greater by the whole ſegment F f than in the firſt caſe; and it is in the caſe of the third hypotheſis leſs by the whole ſegment F *F*, than in the firſt caſe, ſince the ſegments F f and F *F* expreſs the diſtances between the perpendicular F L, and f r and *F R*, conſequently the ſecond hypotheſis furniſhes longer leavers than the other two, and gives it them, even when it is elevated ſufficiently to produce the over-ſetting of the parallelopiped in the two other hypotheſes. Which was to be demonſtrated.

Theſe advantages of the ſecond hypotheſis are ſuch as I have ſet forth in the extract [a] of my mechaniſm. It ſhould be therefore our care to ſecure them to floating bodies from the firſt moment of their inclination. They are ſecured to the parallelopiped.

## FIGURE I.

1°. By making its draught of water F L [b] equal to the root of two thirds of the ſquare of its half breadth.

2°. By whatever cauſes the centre of magnitude r of its part E D O M which is immerged during its inclination to approach [c] ſtill nearer to the plane of its flotation H P when it is erect. In effect the caſe of the ſecond propoſition of §. V. foregoing is the moſt advantageous of all; ſince the quantity F f, whoſe point of ſuſtentation ſhifts towards the ſide B D on which the parallelopiped inclines is ever proportional [d] to the ſegment Z r, which when the parallel a r remains the ſame, is by ſo much greater as the ſegment a Z is leſs. But this laſt is by ſo much leſs, as the centre of magnitude of the part E D O M is leſs diſtant from the flotation H P. Therefore, &c. which was to be demonſtrated.

The centre of magnitude of the immerged part, may be brought ſtill near the plane of flotation, by retrenching ſome of its parts; ſuch for example, as the triangles K D d and M O o. But ſuch retrenchment cannot be made, but at the expence of the capacity of the immerged part, and of the bulk of water which it diſplaces. But the weight of ſuch bulk of water is ever equal to that of the floating body, and conſequently to that of the counterpoiſe which it meets with in its centre of gravity, to oppoſe the forces which produce its inclination: and as it ſtands not leſs in need of a ſtrong counterpoiſe than of an advantageous ſhifting of its point of ſuſtentation, when it is to ſupport great efforts; experience has given the preference to the ſquare pontoon, above all other forms of building, for floating machines uſed in maſting and ſuch as are to produce great efforts: this in effect is that of all built of the ſame length, the ſame breadth, and the ſame weight, which has the centre of magnitude of its immerged part the neareſt to the plane of its flotation, and which inclines the leaſt under the like applications of the ſame forces.

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[a] §. XVIII. Of my extract. [b] Prop. I. of §. IV. Ibid.
[c] Prop. II. of §. V. Ibid. [d] §. VII. Ibid.

An advantageous ſhifting of the point of ſuſtentation may likewiſe be ſecured to the parallelopiped, by every thing which tends to remove from the perpendicular of its point of ſuſtentation when it is erect, the centre of magnitude of that part of it which is immerged during its inclination; ſince the quantity F f whereby the point of ſuſtentation ſhifts towards the ſide B D on which the body inclines is ever proportional to the ſegment Z r [a], and the ſegment a z remaining the ſame, the ſegment Z r is by ſo much the greater, as the parallel a r is greater. But this parallel a r expreſſes the diſtance between the perpendicular L a of the point of ſuſtentation F of the parallelopiped when it is erect, and the centre of magnitude r of its part E D O M which is immerged during the given inclination. Therefore &c. which was to be demonſtrated.

Theſe demonſtrations are equaly applicable to all the caſes of the three hypotheſes, and to all floating bodies.

## §. IX. *Of the girdling of veſſels.*

Experience has ſhewn that by girdling a veſſel about thoſe parts of it which are near the plane of its flotation, its power to carry ſail is increaſed. I here undertake to demonſtrate it by the principles I have eſtabliſhed, which I ſhall apply to the propoſed parallelopiped.

*Let the triangles* B K b *and* S M *s be a girdling applied to the vertical plane* B D O S, *and continued throughout the whole length of the propoſed parallelopiped.*

*The figure* h D O p *its immerged part when it is erect.*

*And the figure* e K D O M *its immerged part during the given inclination.*

As this girdling is continued throughout the whole length of the parallelopiped, it does not hinder the vertical plane b K D O M *s*, from dividing the parallelopiped, during its whole inclination on the ſide B D, into two parts perfectly equal and ſimilar, by which means this vertical plane continues conſtantly to have the ſame centre of magnitude of its immerged part, and the ſame point of ſuſtentation as the parallelopiped; ſo that what I ſhall ſay of this plane, is to be underſtood of the whole parallelopiped.

1°. *The parallel* H P *ſerving for a fixt line.*

I add to the moment of the trapezium E D O M that of the triangle H K h; and from their ſum I ſubſtract the moment of the figure e K D O M, which is immerged during the given inclination: but the moment of the triangle H K h which I add, is a good deal leſs than that of the trapezium E D O M. Moreover the ſurface of the figure e K D O M is greater than that of the trapezium E D O M. The quotient of the moment of ſuch figure e K D O M divided by its ſurface is therefore leſs than the quotient of the moment of the trapezium E D O M divided by its ſurface. But the former of theſe two quotients is equal to the diſtance between the centre of magnitude of the figure e K D O M and the flotation H P; and the ſecond quotient expreſſes the diſtance between the ſaid flotation and the centre of magnitude of the trapezium E D O M: the diſtance a F between the flotation H P; and the centre of magnitude of the part which is immerged during the inclination of the parallelopiped, is therefore diminiſhed by the girdling, the ſhifting of the point of ſuſtentation towards the ſide on which the parallelopiped inclines becomes greater [a], the leaver on which its centre of gravity acts to oppoſe its inclination, and conſequently its power to carry ſail are, proportionally increaſed, which was to be demonſtrated.

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[a] §. VII. Of my extract.

2°. *The perpendicular* L *a ſerving for a fixed line.*

In the given inclination, whether the parallelopiped be girdled or not, the reſpective moments of the trapeziums F L D K and F L O M mutually deſtroy one another and are to be eſtimated $= 0$. So that if it be girdled, the diſtance between the centre of magnitude of its immerged part e KDOM and the perpendicular L *a* is equal to the quotient of the moment of the triangle e F K divided by the ſurface of the figure e KDOM; and if it be not girdled, the diſtance between the centre of magnitude of the immerged part E D O M and the perpendicular L *a* is equal to the quotient of the moment of the triangle E F K divided by the ſurface of the trapezium EDOM. But the former of theſe two diſtances is the greater. In effect, let F H $= a$; F E $= x$; F L $= c$; and e G $= z$; the trapezium E D O M at its ſurface $= 2ac$; and its moment $= \frac{2a^2x}{3}$: The triangle e K E has its ſurface $= xz$; and its moment $= \frac{3axz+xzz}{3}$: Laſtly, the diſtance between the centre of magnitude of the figure e K D O M and the perpendicular L *a* is $= \frac{2a^2x+3axz+xzz}{6ac+3xz}$, whereas the diſtance between this ſame perpendicular and the centre of magnitude of the trapezium EDOM is $= \frac{2ax}{6c}$. I reduce theſe two diſtances to one denomination, and I have the former $= \frac{2a^2cx + 3acxz + cxzz}{6acc43cxz}$ and the ſecond $= \frac{2a^2cx + axxz}{6acc + 3cxz}$: I multiply them by $6acc + 3cxz$; I take from both $2a^2cx$ and divide their remainders by $z$; which gives the firſt $= 3acx + cxz$, and the ſecond $= axx$, and ſhews me that the former of theſe two diſtances is a *maximum*; ſince the part $3acx$ of its remainder is ſingly greater than the whole remainder $axx$ of the latter. In effect the ſine H E of the angle of inclination E F H, expreſſed by $x$ is at the moſt $= c =$ the draught of water F L: if it were greater, the baſe of the parallelopiped would not remain entirely under water; which would be contrary to the hypotheſis, and would require new and different computations to be brought into this memoir, where I have nothing in view but to point out means capable to prevent floating bodies from inclining ſo as to overſet. So that by this ſecond advantage of girdling the parallelopiped, the centre of magnitude of that part of it which is immerged during its given inclination, is brought towards the ſide on which it inclines, the ſhifting of its point of ſuſtentation becomes more advantageous [a], and its power to carry ſail is conſequently increaſed. Which was to be demonſtrated.

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[a] §. VIII. Of my extract.

### EXAMPLE.

Let F H $= 2400$; E H $= 692$: F L $= 2000$; e G $= 30$; its ſurface $= 9600000$; and its moment $= 2657280000$, the triangle e K E has its baſe $= 1384$ its altitude e G $= 30$; its area $= 20760$; its power $= 2410$; and its moment $= 50031600$: laſtly, the figure e KDOM has its ſurface $= 9620760$, and its moment $= 2707311160$, ſo that the diſtance between the centre of magnitude of this figure and the perpendicular L$a$ is $= \frac{2707311600}{9620760} = \frac{112804650}{400865}$; whereas that between this ſame perpendicular and the centre of magnitude of the trapezium EDOM is $= \frac{26572800}{9600} = \frac{110959432}{400865}$. The former is therefore greater than the latter by a quantity $= \frac{1845218}{40005}$, and yet the girdling propoſed in the example is very moderate. Therefore, &c. which was to be demonſtrated.

As for ſuch as are willing to cary their inquiries farther, and make a computation of the whole quantity whereby the centre of magnitude of the part which is immerged during the given inclination, is by the girdling brought towards the ſide on which the parallelopiped inclines, I ſhall put them into the way of it, by admoniſhing them.

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[a] §. VIII. Of my extract.

1°. That by ſuppoſing $FH = a$; $EH = x$; $hH = n$; and $EG = y$, the ſimilar triangles h K H, and e K G, give the parallel $eG = \frac{2nx + ny}{x}$. The triangles F E H and e G E, which are likewiſe ſimilar, give the ſame parallel $eG = \frac{ay}{x}$. So that by means of this equation we have $EG = \frac{2nx}{a-n}$, and the parallel e G ( = 3[a]) $= \frac{2an}{a-n}$. 2°. That by ſuppoſing $FH = 2400$; $EH = 692$; and $hH = 14\frac{146}{161}$ we have the ſegment $EG = 8\frac{13}{29}$, and the parallel $eG = 30$. 3°. That the parallel H P ſerving for a fixed line, the reſpective moments of the triangles h E H and h K H mutually deſtroy each other, and are to be eſtimated $= 0$: the triangle E h N has its baſe E N double to the ſegment h H, and its power equal to two thirds of its altitude E H; and the triangle e N E has its baſe = N E; and its power equal to the ſum of the ſegment F H and of one third of its altitude G e.

As to the reſt, theſe truths and their demonſtrations hold good in all manner of floating bodies, and in ſuch eſpecially as are to be built for the ſea: but notwithſtanding this two-fold advantage of girdling veſſels, as it conſiderably abates their wake, and increaſes their lee way, yet we cannot too much avoid coming into it, and ſtudy to give them ſuch a form as that they may not require it. Now that I have laid down the laws of the ſhifting of the point of ſuſtentation, the geometricans and builders will be eaſily led into the proper ſhape to anſwer this end: nor will it leſs contribute to abate the rowling, and the dip of ſhips than to increaſe their power to carry ſail.

### §. X. *A continuation of the obſervations on the mechaniſm of the motions of a floating rectangular parallelopiped.*

If, according to the experiments which I have propoſed in my extract, and cauſed to be put in execution; the two gudgeons by which it is ſuſpended in air, remain fixed in the middle of its length, its inclinations on its length, in the caſes of the two laſt hypotheſes, ſhould be in air different from what they are in water, under the ſame applications of the ſame forces. The demonſtrations which I have given of the ſhifting of its point of ſuſtentation in water, and the details which I have gone into in my extract, can leave no manner of doubt of this truth.

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[a] Demonſtration foregoing.

## FIGURE II.

If during an inclination of the parallelogram produced by a force Œ, whoſe direction Œ A A is horizontal, I apply to the ſides of its actual flotation, two horizontal iron pins, ſo as that they may prolong the axis of its inclination as I have determined; and if I ſubject them at the ſame time to the ſame ſhiftings as axis, I ſhall be aſſured, that upon ſuſpending the parallelopiped in air by theſe two pins, it will there acquire the ſame inclinations as in water, upon the ſame applications of the ſame forces; becauſe its axis of inclination will be the ſame in air as in water, and the leaver of the force O E will, in both theſe fluids, be equal to the diſtance A A Ψ between its direction O E A A and the plane e m of the actual flotation of the parallelopiped. It is the ſame if the given inclination is produced by the horizontal force W, whoſe leaver, is in air as well as in water equal to the diſtance A V Ω between its direction W A V and the flotation of the parallelopiped e m.: which manifeſtly proves the ſlightneſs of the objection which I have mentioned, and the ſolidity of the refutation which I have given thereof in §. XX. of my extract.

In a word, of what kind ſoever are the forces applied to the floating parallelopiped, whatever are the inclinations which they give it; apply two gudgeons to the extremities of its axis of inclination, as I have determined; ſubject them to the like ſhiftings as the ſame axis undergoes when it is a-float: whether you ſuffer it to remain there or ſuſpend it in air by its gudgeons, its inclinations in the air and in the water will be the ſame, under the ſame applications of the ſame forces, in both fluids, and ſuch as I have determined in my extract [a], and you will perceive the laws of their mechaniſm in their different motions, to be ſuch as I have explained [b].

## FIGURE I.

*Let the points r R and R be different places of the centre of gravity which is common to the heavy parts of the parallelopiped propoſed, and to the forces applied thereto, in the three hypotheſes of the ſhifting of its point of ſuſtentation.*

If at the inſtant that one or more external forces are applied to it, and whilſt it ſtill remains erect, I draw a line through the centre of

[a] §. IV, V, VII, and VIII. Of my extract.

[b] §. IX, X, and XI. Of my extract.

gravity which is common to its heavy parts, and to the forces which are applied to it, and through the point where its point of ſuſtentation will be in the inclination produced by ſuch forces; the angle of incidence of that line, or the perpendicular to the horizon which then paſſes through the ſame point of the future ſuſtentation, is ever equal to that of the inclination which theſe forces produce. Thus the angle of its inclination expreſſed by the angle E F H is, in the caſe of the firſt hypotheſis, equal to the angle R F L; in the caſe of the ſecond hypotheſis, equal to the angle r f X; and in the caſe of the third hypotheſis, equal to the angle *R F* Y; becauſe the angle E F H of the given inclination, is equal to that of the incidence of H P, which expreſſes its flotation when it is erect, upon E M which expreſſes its flotation in ſuch given inclination. The lines F L, f X, and *F* Y are to be ſuppoſed perpendiculars to H P; the lines F R, f r, and *F R* perpendiculars to E M, and that of theſe latter, wherein the common centre of gravity reſides, to be in the perpendicular of the point of actual ſuſtentation of the parallelopiped, during the ſame given inclination.

## FIGURE II.

Likewiſe if during a given inclination, I draw the line g F from the point g, where the centre of gravity of the parallelopiped reſides, to the point E, where its point of ſuſtentation will be when it becomes erect, the angle g F L of incidence of the ſaid line g F upon F L perpendicular to the horizon, which paſſes then through the future point of ſuſtentation F, is, for the reaſons which I have alledged, equal to the angle E F H of the actual inclination.

The like practical operations applied to all manner of floating bodies, and to all ſuch as are ſucceſſively ſuſpended in air, by different points below their ſummits, do equally furniſh the knowledge of the angles of their inclinations.

Finally, the ſeveral truths which I have diſcloſed, with their demoſtrations, are common to all floating bodies; and I ſhall entertain infinite hopes of the ſucceſs of this memoir, if it ſhall be though worthy the attention of ſuch geometricians and builders as have been pleaſed to requeſt it of me: if it impreſſes on them a due ſenſibility of my diſcovery of the mechaniſm of the motions of floating bodies; if, in a word, if gives them an idea of the ſolidity and the ſimplicity of the theory of that mechaniſm, its perfect agreement with all poſſible experiments, and the certainty and uſefulneſs of its practice.

*The END.*

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Fig. I

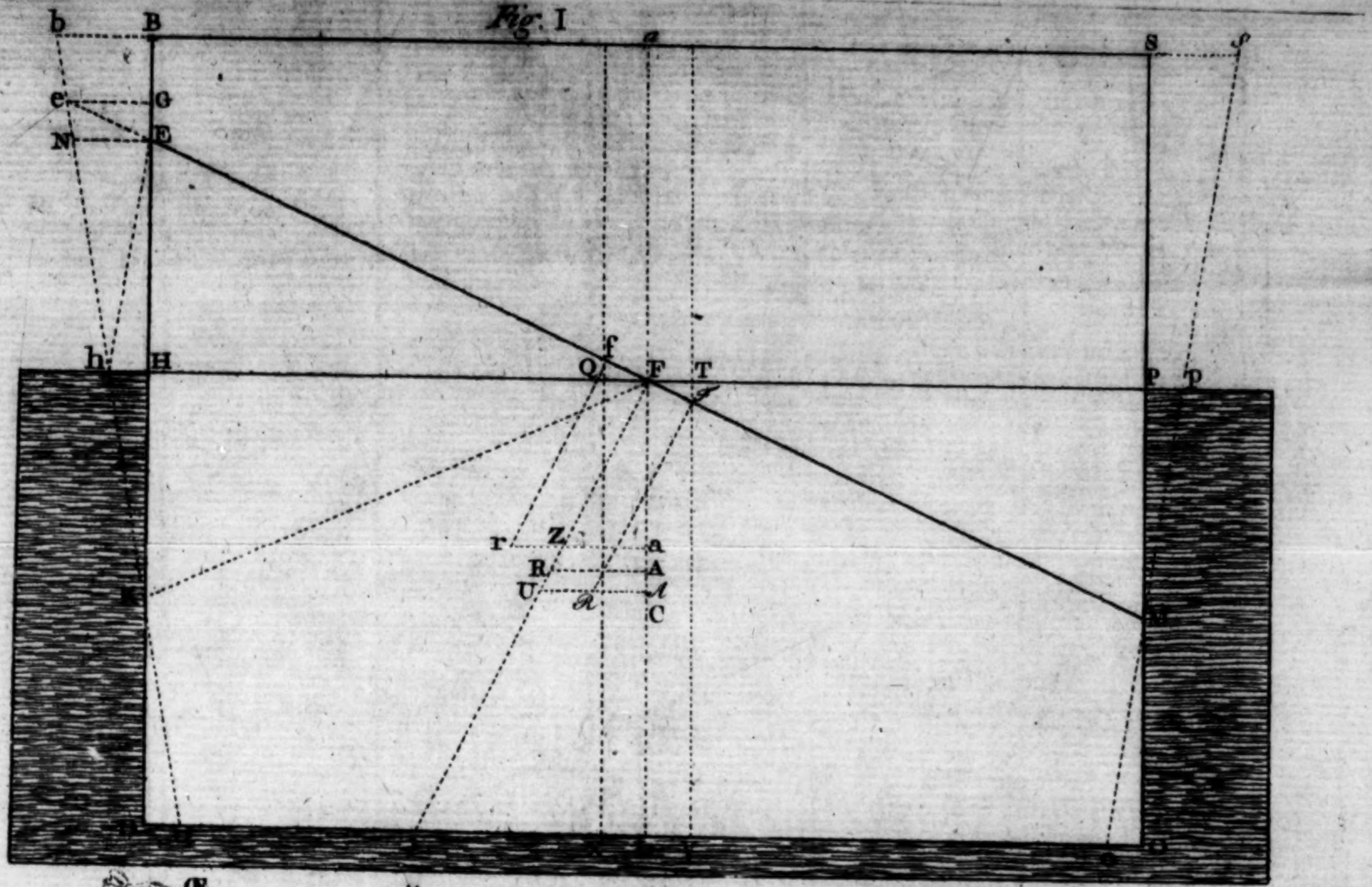
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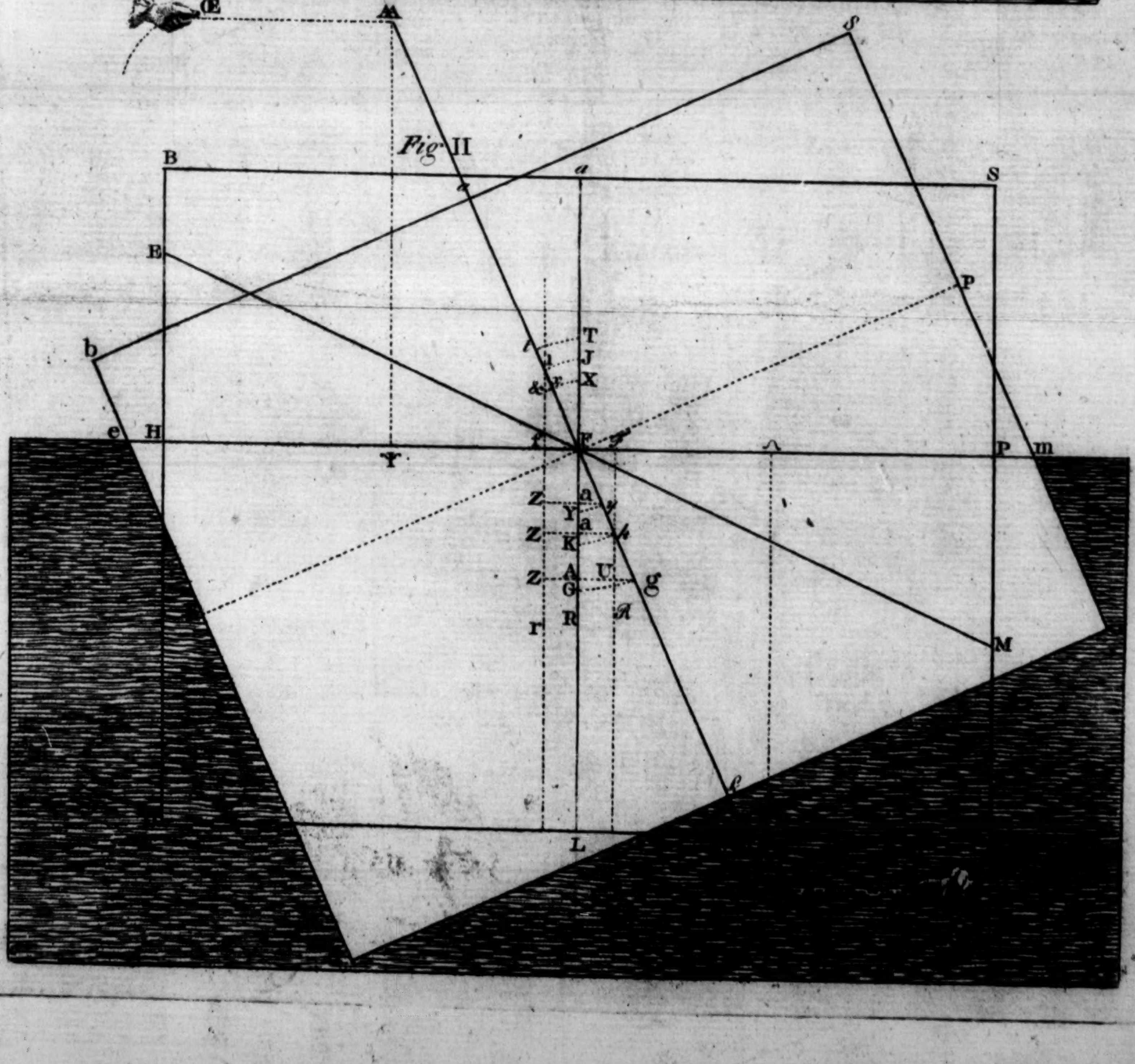
Fig. II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